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ON THE CLUSTERING TENDENCIES AMONG THE NEBULAE

II. A STUDY OF ENCOUNTERS BETWEEN LABORATORY MODELS OF STELLAR SYSTEMS BY A NEW INTEGRATION PROCEDURE

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ABSTRACT

In a previous paper[1] the writer discussed the possibility of explaining the observed clustering effects among extragalactic nebulae as a result of captures. The present investigation deals with the important problem of whether the loss of energy resulting from the tidal disturbances at a close encounter between two nebulae is large enough to effect a capture. The tidal deformations of two models of stellar systems, passing each other at a small distance, are studied by reconstructing, piece by piece, the orbits described by the individual mass elements. The difficulty of integrating the total gravitational force acting upon a certain element at a certain point of time is solved by replacing gravitation by light. The mass elements are represented by light-bulbs, the candle power being proportional to mass, and the total light is measured by a photocell (Fig. 1). The nebulae are assumed to have a flattened shape, and each is represented by 37 light-bulbs. It is found that the tidal deformations cause an increase in the attraction between the two objects, the increase reaching its maximum value when the nebulae are separating, i.e., after the passage. The resulting loss of energy (Fig. 6) is comparatively large and may, in favorable cases, effect a capture. The spiral arms developing during the encounter (Figs. 4) represent an interesting by-product of the investigation. The direction of the arms depends on the direction of rotation of the nebulae with respect to the direction of their space motions.

I. THE EXPERIMENTAL ARRANGEMENTS

The present paper is a study of the tidal disturbances appearing in stellar systems which pass one another at small distances. These tidal disturbances are of some importance since they are accompanied by a loss of energy which may result in a capture between the two objects. In a previous paper[1] the writer discussed the clustering tendencies among extragalactic nebulae. A theory was put forth that the observed clustering effects are the result of captures between individual nebulae. The capture theory seems to be able to account not only for double and multiple nebulae but also for the large extragalactic clusters. The present investigation tries to give an answer to the important question of whether the loss of energy accompanying a close encounter between two nebulae is large enough to effect a capture.

A study of tidal disturbances is greatly facilitated if it can be restricted to only two dimensions, i.e., to nebulae of a flattened shape, the principal planes of which coincide with the plane of their hyperbolic orbits. In order to reconstruct the orbit described by

[1] *Mt. W. Contr.*, No. 633; *Ap. J.*, **92**, 200, 1940.

a certain mass element belonging to one of the two nebulae, we must first derive as a function of the time the x and y components of the total gravitational force acting upon the element. Starting from a certain distribution of mass in the nebulae, we may find the total gravitation by a purely numerical integration. However, such an integration is impracticable on account of the large amount of work involved. In the present case a solution has been found by replacing gravitation by light. Every mass element is represented by a small light-bulb, the light being proportional to the mass, and the total light along the x and y axes is measured by a combination of a photocell and a galvanometer. The measured values represent the components of the gravitational force. The latter components are obtained by adding up the attractions due to individual mass elements, each multiplied by the cosine of the corresponding projection angle. Consequently, the photocell must obey the cosine law as far as the angle of incidence of the light is concerned. If the photocell obeys the cosine law and if the combination of photocell and galvanometer gives a linear relation between light and scale reading, the galvanometer deflection will be proportional to the total gravitational force or, more correctly, to the total acceleration. A detailed account of the instrumental arrangement is given below.

The light-bulbs, which represent the individual mass elements, must fulfil three different requirements. Their candle powers must be the same within certain limits. Furthermore, the candle power must not change when the light-bulb is rotated about its axis of symmetry. The third condition is that the light must not show any appreciable decrease with time. The light-bulbs used in the present case were designed and manufactured by the Luma Factory, Stockholm, Sweden. Figure 1 shows a cross-section of the lamp. The vertical spiral filament is situated at the center of a spherical glass bulb with a rough inner surface. For 100 light-bulbs the difference in candle power is found to be within 10 per cent, whereas the deviations corresponding to rotation about the axis of symmetry average ± 2.1 per cent. Since differences of the first kind can be largely eliminated by using various resistances, the above results are quite satisfactory. The spiral filament was designed for a current of 3.8 volts and 0.3 amperes. The use of a voltage of less than 2 volts insures constancy of the light over a long period.

Each light-bulb is mounted on a steel needle as shown in Figure 1. The plane surface on which the experiments are performed consists of a layer of cardboard covered by a layer of brass plate (thickness, 0.05 mm) and a layer of thin black paper. The brass plate is connected to one pole of the battery, the other pole being connected with a single, isolated wire leading to each light-bulb. In this way the number of wires connected with each lamp is reduced from two to one, which greatly facilitates the shifting of the lamps during the measurements. The black paper, which was especially made for this investigation, is covered by a co-ordinate system of faint gray lines. The reflection coefficient is small and will be further discussed below.

The photocell that is used for the measurements is shown in half-size in Figure 1. The cell is of the special type called "Sperrschichtphotometer." The incident light penetrates a thin layer of Cu and is then absorbed in a thick layer of Cu_2O mounted on a Cu-plate.[2] The latter plate forms the positive pole of the element, while the negative pole is represented by the thin Cu-layer. It was pointed out above that the photocell must obey the cosine law with regard to the angle of incidence of the light. In Figure 2 the full curve gives the measured values, while the dotted line represents the theoretical cosine law. The agreement is quite satisfactory for the present purpose.

The galvanometer was manufactured by the Leeds and Northrup Company of Philadelphia. The instrument is of the light-pointer type and has a sensitivity of about 0.025 microampere per millimeter scale division. The moving coil has a resistance of 1000 ohms. During the measurements the galvanometer is connected with the photocell through a variable resistance of 10,000 ohms. Figure 2 shows the relation between light-

[2] Cf. R. Sewig, *Objective Photometrie*, p. 28, 1935.

intensity and galvanometer deflection. The two lines correspond, respectively, to the limiting cases of 0 and 10,000 ohms of additional resistance. The deviations from straight-line relations are insignificant.

Before proceeding further we will call attention to some disturbing reflection and obscuration effects that affect the measurements. Light from each lamp is reflected by the black-paper surface and by other lamps. On the other hand, part of the light may be cut off from the photocell by other lamps. The proportion of light reflected by the black-paper surface is found by experiment to be approximately independent of the distance between light-bulb and photocell and to amount to about 5 per cent. The constancy of the reflection effect is nicely demonstrated by the relations between light-intensity and galvanometer deflection that are given in Figure 2. The different light-intensities are obtained by placing the same light-bulb at different distances from the photocell, the distance ranging from 30 to 120 cm. Although the reflections are included in the corresponding light-intensities, the deviations from a straight-line relation are very small.

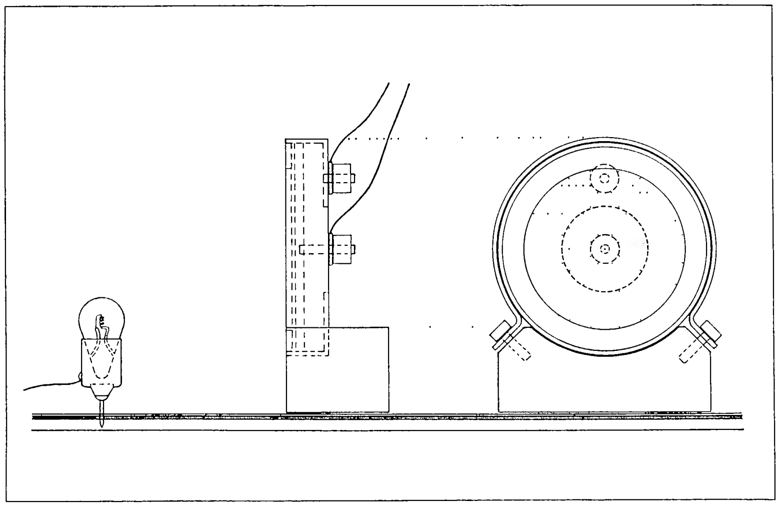

FIG. 1.—Cross-section of light-bulb and photocell (half-size)

Similarly, the reflections by the other lamps will not cause any appreciable disturbance. The diameter of a light-bulb is 8 mm, a small value compared with the distances between the different lamps, which range from 10 to 20 cm. The obscuration effect is largely eliminated by mounting the photocell as shown in Figure 1. The center of the sensitive surface is raised 2 cm above the lamp centers. The maximum obscuration should amount to only a few per cent.

A schematic picture of the arrangement of light bulbs, batteries, and other accessories is given in Figure 3. The lamps are arranged in two groups, A and B, each containing 37 lamps and having a diameter of 80 cm. Each group represents a nebula. By using three or more different voltages it is possible to reproduce, within certain limits, any desired distribution of mass. In order to obtain more complicated distributions it may be necessary to use a larger number of lamps. In the present case, however, the simplest possible assumption of the internal mass distribution is made. For practical reasons the number of light-bulbs should be kept as small as possible. The battery is composed of thirty 2-volt accumulators, connected in parallel. For a total of 74 lamps, arranged in parallel, the discharge amounts to about 12 amperes. In order to stabilize the voltage, the battery is continuously recharged by about 6 amperes from the municipal-power network of 120 volts. Since the candle power of a light-bulb is approximately proportional to the third power of the voltage, it is important to keep the voltage as constant as possible. In order to calibrate the photocell and to check minor variations of the voltage, a control device consisting of 5 additional light-bulbs was constructed. To make sure of the same temperature and the same external conditions, the control device is kept in the room where the measurements are made. A possible decrease of light with time is neutralized by always having the control lamps burning simultaneously with the other 74 lamps. Checks are frequently made by placing the photocell in a special mounting at a fixed distance from the 5 control lamps. Minor deviations in the control reading of the galvanometer are neutralized by changing the variable resistance connecting the photocell and galvanometer.

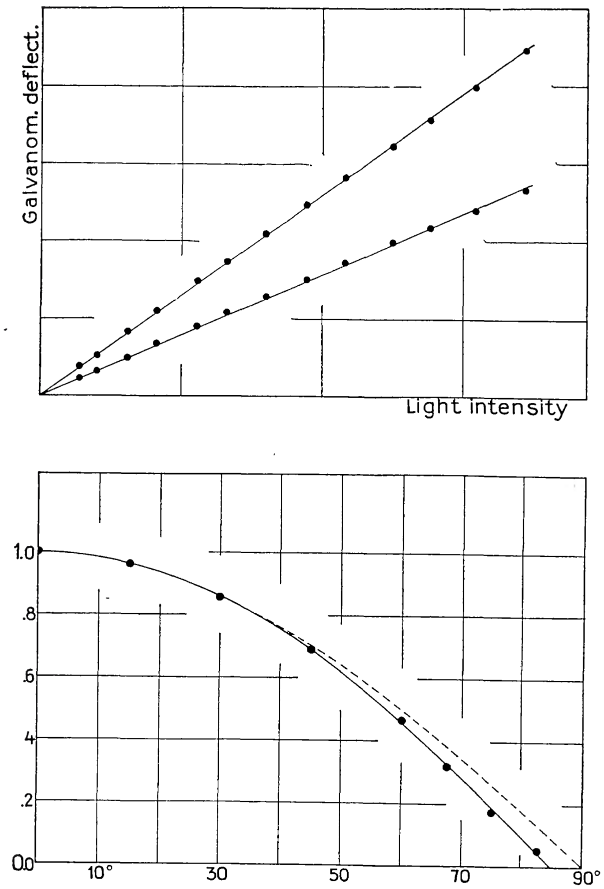

FIG. 2

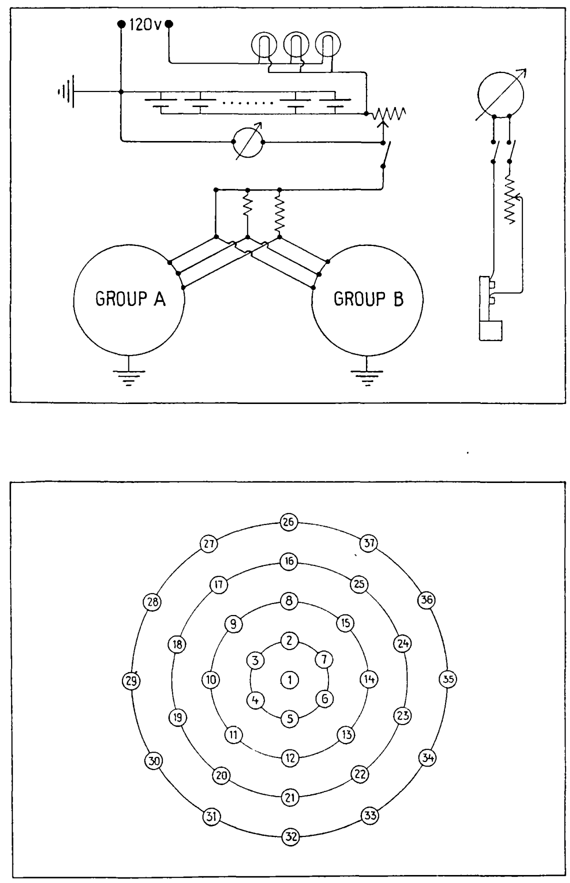

FIG. 3

FIG. 2.—(*a*) Relation between galvanometer deflection and light-intensity (resistance of 0 and 10,000 ohms). (*b*) Relation between galvanometer deflection and angle of incidence of light (*dotted line* = theoretical cosine law).

FIG. 3.—(*a*) Coupling scheme. (*b*) Arrangement of the 37 light-bulbs in groups A and B

II. THE MEASUREMENTS

Before starting the measurements, we must make some assumption regarding the internal distribution of mass in the nebulae. An indication of the mass distribution is obtained from studies of the distribution of light in nebulae, which show that the surface brightness increases rapidly toward the center. Although the assumption of a strict proportionality between mass and light may not be justified,[3] it seems permissible to con-

[3] Cf. H. W. Babcock, *Lick Obs. Bull.*, **19**, 41, 1939.

clude that the density of mass generally increases toward the center of a nebula. In the absence of any detailed knowledge the simplest assumption with regard to mass distribution is probably a normal error-curve. In the present case the mass distribution will enter the problem mainly as a statistical quantity, and certain deviations from the assumed distribution will not materially change the final results. A normal distribution of light is obtained by giving lamps 1–15 (see Fig. 3) a relative candle power of 1.0, lamps 16–25 a power of 0.7, and lamps 26–37 a power of 0.3.

It should be pointed out that the numerical values adopted for the total mass and the absolute diameter of the nebulae are of comparatively small importance in the determination of tidal deformations. In the case of parabolic motions the space velocities and internal rotational velocities will both be proportional to the square root of the mass. Thus a change in mass will have the same effect as a change in the unit of time. This is approximately true even for hyperbolic motions, if the initial space velocities are not too large. Similarly, the parabolic and the rotational velocities are inversely proportional to the square root of the diameter, if the diameter is used as a unit of distance. In the cases where numerical values are desirable, the mass has been put equal to 10^{11} solar units and the diameter of 2500 parsecs.[4] However, only the ratio between the two values needs to be fixed.

As has been pointed out above, the measurements are performed on a plane surface covered by a layer of black paper with an imprinted co-ordinate system. The dimensions of the surface are about 300×400 cm. At the beginning the two groups of lamps, each representing a nebula, are assembled at either end of the surface. The diameter of each group is 80 cm. By means of different resistances, the candle powers of the light-bulbs are adjusted in order to fit a normal distribution of light, as has been described above. The different lamps are then moved step by step, their orbits describing the space motion of the nebula combined with the internal rotation. The two rotating systems will gradually approach each other. As to the rotation, the mass elements are supposed to move in the same direction in circular orbits about the center of the nebula. The space motions depend on the initial velocities (at infinite distance) of the two objects and on the distance of closest approach. All motions are assumed to be undisturbed by possible clouds of dark material inside or outside the nebulae. In order to obtain a complete picture of the tidal disturbances accompanying a close encounter, we must make different assumptions concerning the three parameters represented by (1) the direction of rotation, (2) the initial velocity, and (3) the distance of closest approach. The dependence on the first parameter has been studied by undertaking two series of measurements corresponding to clockwise and counterclockwise rotations, respectively. With regard to initial velocity, two different values have been selected, namely, 0 (parabolic motion) and 450 km/sec. In view of the fact that the average space velocity of nebulae outside the large clusters amounts to about 300 km/sec, as indicated by observed radial motions, these values may represent limiting cases.[5] Finally, the distance of closest approach has been varied. The main efforts, however, have been concentrated on the case where the

[4] By using the apparent major diameters and magnitudes given in the Shapley-Ames catalogue of bright nebulae and by adopting an average, absolute magnitude, $\overline{M}_m$, equal to -15.2 (cf. Hubble, *Mt. W. Contr.*, No. 548; *Ap. J.*, **84**, 158, 1936), we derive the following average values of log D (D=absolute diameter): 3.01 (type E), 3.25 (type Sa), 3.37 (type Sb), and 3.40 (type Sc). Thus a diameter of about 2500 parsecs is obtained for the intermediate and late-type spirals. It may be remarked that the diameters to be used in the present case should refer to the main body of the nebulae, since the faint, outermost parts have a small mass density.

[5] If there were no selection effects, the average value of the initial velocity, V_0, would be equal to about two-thirds of the average space velocity. However, the frequency of encounters between a certain nebula and other objects is proportional to the relative velocity of this nebula with respect to the others. Owing to this selection effect, the average value of V_0 will amount to about 75 per cent of the average space velocity, or approximately 225 km/sec.

two nebulae pass each other edge to edge (closest distance between centers = diameter of nebula).

The orbits of individual mass elements, i.e., lamps, are constructed piece by piece, by joining the orbital elements, each corresponding to a certain interval of time. For practical reasons the elements should have a certain length, neither too small nor too large, which necessitates the definition of a proper unit of time. It is found that the interval of time corresponding to a rotational motion at the edge of a nebula of 12 cm would be a suitable unit. Knowing the motion of a mass element up to a certain point of time, we are able to construct the piece of orbit corresponding to the subsequent unit of time if the x and y components of the acceleration caused by the total gravitational force acting upon the element can be determined. The accelerations are measured by the photocell. The lamp is removed and replaced by the photocell, which is turned successively in the directions $+x$, $-x$, $+y$, and $-y$. The galvanometer is calibrated by comparing the computed centripetal accelerations in one of the nebulae with the corresponding measured values. The resistance connecting the photocell and the galvanometer is adjusted in such a way as to make the galvanometer reading equal to the acceleration in millimeters (per square unit of time).

The constructions and drawings of the orbits of the individual mass elements are made on large sheets of graph paper. Since the two nebulae are identical, it is necessary to consider only one of the objects. The center of the nebula is kept fixed. Instead, the point representing the center of mass of the two objects is moved in a parabolic, or hyperbolic, orbit. The greatest possible care is exercised in drawing the tangents of the individual curves and in giving the proper curvature to every additional piece of orbit. A set of French curves is used for this purpose. Despite the care taken, the derived orbits will, of course, give only an approximate picture of the real motions inside the nebula. However, the accuracy ought to be sufficient to give a fair idea of the general tidal deformations and the loss of energy accompanying a close encounter.

III. THE RESULTS

Regarding the tidal deformations of the two nebulae, we refer at once to Figures 4*a* and 4*b*, which give a good illustration of the average tidal disturbances. The figures correspond to the case where the objects meet edge to edge, i.e., the distance of closest approach equals the diameter of the nebulae. In Figure 4*a* the objects have a clockwise rotation, whereas Figure 4*b* corresponds to rotations in the opposite direction. In both cases the motions are parabolic, i.e., the initial space velocities are equal to zero. However the figures also represent the deformations of nebulae moving in hyperbolas with small eccentricities, since the changes corresponding to initial velocities of 100–200 km/sec are rather small. The common mass center is denoted by a cross. It appears that the deformations are rather small before the nebulae have reached the distance of closest approach. The maximum deformation occurs after the passage. Thus the tidal deformation, if expressed as a function of time, is an asymmetrical effect. Furthermore, the tidal effects are asymmetrical in space, the deformations being larger on those sides of the nebulae that are turned toward one another. These asymmetries in time and space, which are further discussed below, represent two conditions which must be fulfilled in order that a capture may take place.

The most striking features of Figures 4 are the spiral arms which gradually develop during and after the passage. In the case of clockwise rotations the arms point in the direction of rotation, whereas the opposite is true when the rotations are counterclockwise. This result is very interesting in the light of past and present discussions about the direction of spiral arms. Arguments, theoretical and observational, have been put forth for arms pointing in both directions. The above mechanism for the "ejection" of spiral arms suggests that their direction depends on the direction of rotation of the nebulae compared to the direction of the relative space velocity. It should be pointed out that,

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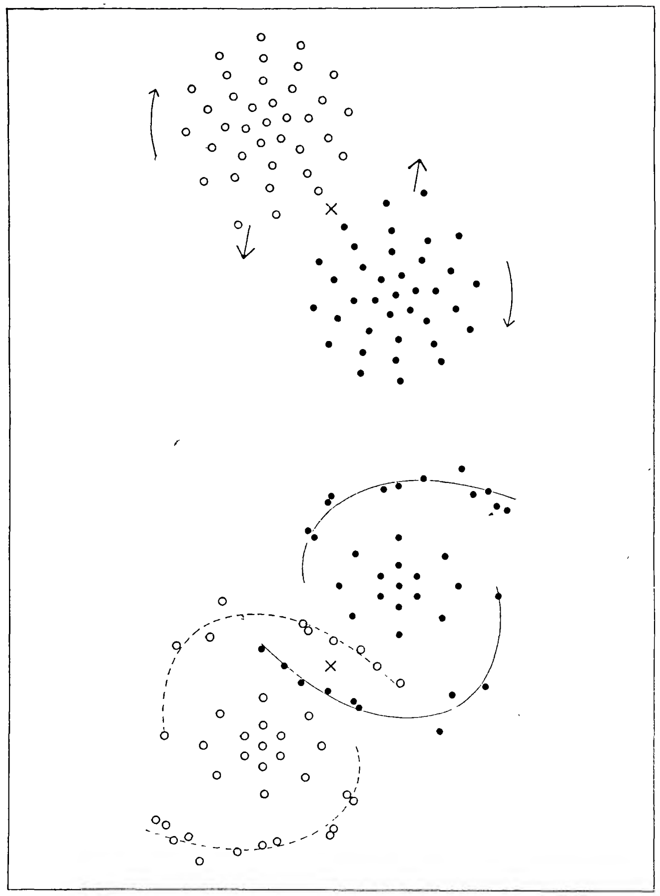

FIG. 4*a*

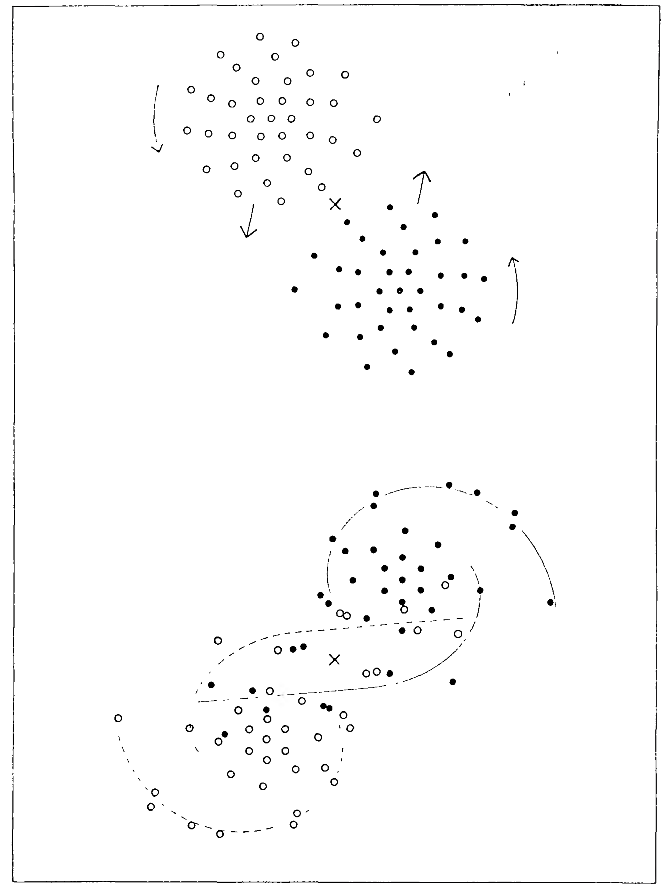

FIG. 4*b*

FIG. 4*a*.—Tidal deformations corresponding to parabolic motions, clockwise rotations, and a distance of closest approach equal to the diameters of the nebulae. The spiral arms point in the direction of the rotation.

FIG. 4*b*.—Same as above, with the exception of counterclockwise rotations. The spiral arms point in the direction opposite to the rotation.

whatever the direction of the arms, the ejected matter rotates in the same direction as the main body of the nebula. The observed spiral arm represents merely the distribution in space of this material. On account of the approximations and the small number of mass elements used, it is not possible in the present case to obtain any conclusive evidence with regard to the form of the arms and the distribution of matter within them. According to observations, most spiral arms seem to have the approximate form of a logarithmic spiral. The position of the points of ejection represents another important

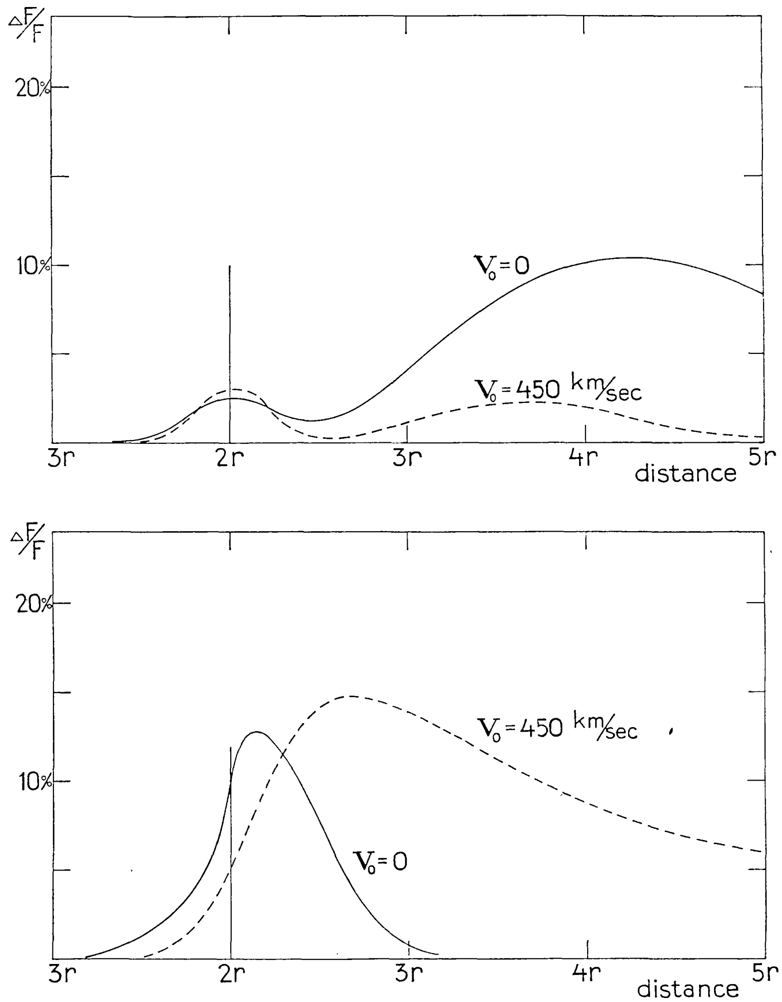

FIG. 5.—Relative increase of attraction between the nebulae (caused by tidal deformations) as a function of the distance between the centers. The distance of closest approach equals the diameter ($=2r$) of the nebulae. The rotations are clockwise (*upper figure*) and counterclockwise (*lower figure*).

problem. It has not yet been possible to decide whether these points remain fixed in space or rotate with the nebula.

On account of the above-mentioned asymmetry in space, the tidal deformations will cause an increase in the gravitational attraction between the two nebulae. In Figure 5 the relative increase is given as a function of the distance between the centers of the nebulae, expressed in terms of the radius. The distance of closest approach equals the diameter of the nebulae. The two curves correspond to initial velocities of 0 (parabolic motion) and 450 km/sec, respectively. The asymmetry of the curves, with respect to the distance of closest approach ($2r$), is very conspicuous. The increase in the attraction amounts to as much as 10–15 per cent. In the case of clockwise rotations the parabolic motion gives a larger increase than the hyperbolic one. This is quite natural, since the slower motion should correspond to larger disturbances. A peculiarity is found in the two maxima, corresponding to distances of about $2r$ and $4r$, respectively. The maxima are caused by the same bulk of ejected matter and are separated by an interval of time corresponding roughly to one full rotation of the nebulae. In the case of counterclock-

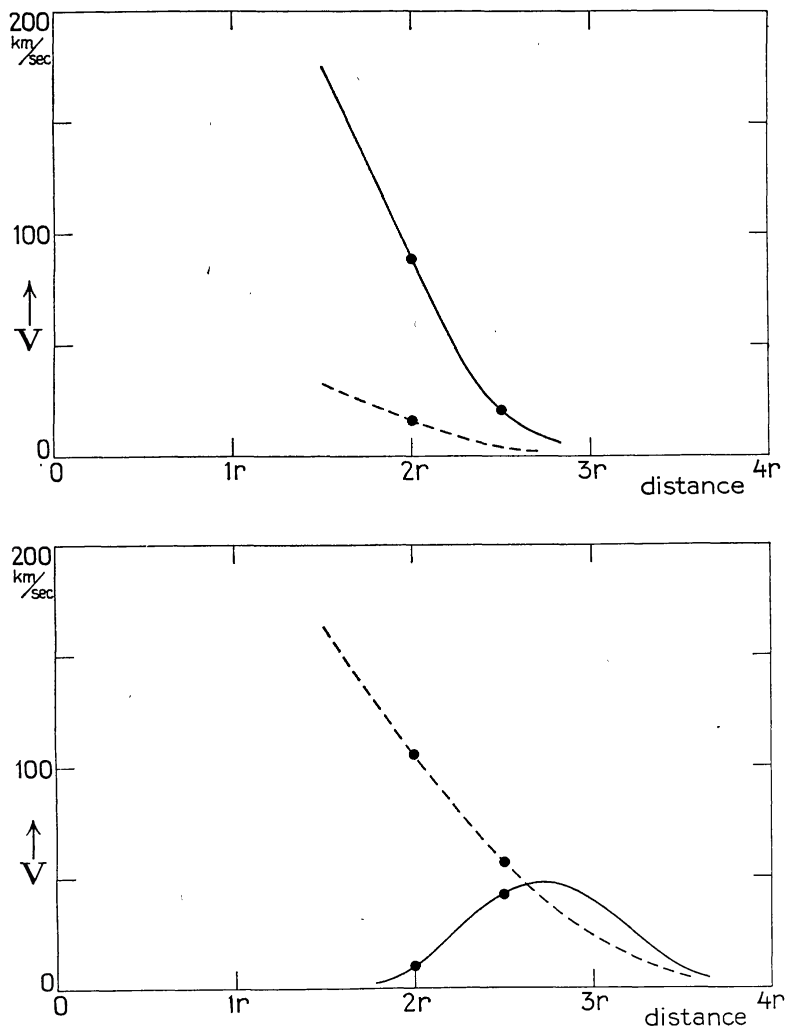

FIG. 6.—Total loss of energy (represented by the velocity V instead of the expression $MV^2/2$) as a function of the distance of closest approach (expressed in terms of the radii of the nebulae). The different curves correspond to those of Figure 5.

wise rotations the parabolic motion also corresponds to larger disturbances than the hyperbolic motion. However, the deformations in this case will be so large that individual mass elements belonging to each nebula are captured by the other, resulting in a decrease in the attraction. The exchange of matter is larger for parabolic motion. This explains the peculiar forms of the two attraction-curves corresponding to counterclockwise rotation.

Having derived the curves giving the increase in attraction, we are able to determine the total loss of energy accompanying a close encounter. In Figure 6 the loss of energy is given as a function of the distance of closest approach between the nebulae, expressed in terms of the radius of the objects. The ordinate represents the velocity V, instead of the expression $MV^2/2$, the decrease in velocity being a more convenient measure of the loss of energy. The different curves correspond to those of Figure 5, i.e., parabolic motion is represented by full curves and hyperbolic motion ($V_0 = 450$ km/sec) by dotted lines. Numerical values are given for distances of $2r$ and $2.5r$. The curves are drawn in such a way as to give the most probable relation between velocity V and distance. The values have been computed by a process of numerical integration. An additional attraction, corresponding to a certain point of time, results in an additional acceleration of the nebula, directed toward the other nebula. By summing up all additional accelerations in x and y, corresponding to the whole interval of time of the encounter, a final velocity (approximately opposite to the space motion of the nebula) is derived that is representative of the total loss of energy. It should be pointed out that the numerical values corresponding to a distance of $2.5r$ are somewhat uncertain. The tidal effects in these cases are rather small, and the deformations are more or less comparable to the errors of measurement and the uncertainties in the construction of the orbits of individual mass elements. However, the principal results of the present investigation depend mainly on the order of size of the computed numerical values. No attempt has been made to find the loss of energy corresponding to encounters at smaller distances than $1.5r$. In such cases the two nebulae will be extensively intermingled, and complications may arise that cannot be properly reproduced by the simple experimental arrangements used in the present case.

In the case of clockwise rotations of the nebulae it appears from Figure 6 that the parabolic motion gives a considerably larger loss of energy than the hyperbolic motion corresponding to an initial velocity of 450 km/sec. It should be remarked that the full curve gives approximately the loss of energy even for nebulae moving in hyperbolas with small eccentricities, since the changes corresponding to values of V_0 equal to 100–200 km/sec are rather small. The loss of energy increases rapidly when the distance of closest approach becomes smaller. The full curve gives an increase in V from 20 to about 175 km/sec when the distance changes from $2.5r$ to $1.5r$. This means that a capture will take place if the initial velocities of the two nebulae are smaller than these values. The loss of energy given by the dotted line is very small. Since the velocity V does not reach the initial value of 450 km/sec, no capture will take place. In the case of counterclockwise rotations the hyperbolic motion generally gives a larger loss of energy than the parabolic motion. This is explained by the peculiar form of the corresponding attraction-curves given in Figure 5. However, the hyperbolic motion will not result in a capture, since the maximum value of V given by the dotted curve is below 450 km/sec. The curve corresponding to parabolic motion shows a maximum for a distance approximately equal to $3r$. This depends on the exchange of matter between the two nebulae, the exchange being smaller when the distance of closest approach is larger. The largest value of initial velocity that corresponds to a capture equals about 50 km/sec.

The numerical values given in Figure 6 depend to some extent on the mass and diameter of the nebulae. In the case of parabolic motions the velocity V is proportional to the square root of the total mass and inversely proportional to the square root of the absolute diameter. In the case of hyperbolic motions the dependence is somewhat smaller. The numerical values used in the present case are 10^{11} solar masses and 2500 parsecs, respectively. However, only the ratio between the two values needs to be fixed. If the diameter is assumed to be equal to 1000 parsecs, the curves of Figure 6 will correspond to a mass of $4 \cdot 10^{10}$ solar units.

The above results are, of course, subject to a certain degree of uncertainty, depending on errors involved in the measurements and on difficulties in constructing exactly the orbits of individual mass elements. Furthermore, the results depend on the special assumptions made in the present case about orientation of the nebulae and internal distribution of mass. In spite of these restrictions it seems justified to conclude that the numerical values derived above indicate the proper order of size of the energy loss in a close encounter between extragalactic nebulae and that, in favorable cases, captures may occur.

The laboratory work was performed at the Lund University Observatory. The author wishes to express his sincere thanks to Dr. Knut Lundmark for generous economic support in procuring and constructing the experimental equipment. The author also expresses his gratitude to the Luma Factory of Stockholm for valuable help in designing and manufacturing the special light-bulbs that were used in the present investigation.

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