THE TACOMA NARROWS BRIDGE FAILURE
Revision A

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April 7, 2009

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Introduction

The original Tacoma Narrows Bridge was opened to traffic on July 1, 1940. It was located in Washington State, near Puget Sound.

The Tacoma Narrows Bridge was the third-longest suspension bridge in the United States at the time, with a length of 5939 feet including approaches. Its two supporting towers were 425 feet high. The towers were 2800 feet apart.

Design

Prior to this time, most bridge designs were based on trusses, arches, and cantilevers to support heavy freight trains. Automobiles were obviously much lighter. Suspension bridges were both more elegant and economical than railway bridges. Thus the suspension design became favored for automobile traffic. Unfortunately, engineers did not fully understand the forces acting upon bridges. Neither did they understand the response of the suspension bridge design to these poorly understood forces.

Furthermore, the Tacoma Narrows Bridge was built with shallow plate girders instead of the deep stiffening trusses of railway bridges. Note that the wind can pass through trusses. Plate girders, on the other hand, present an obstacle to the wind.

As a result of its design, the Tacoma Narrows Bridge experienced rolling undulations which were driven by the wind. It thus acquired the nickname "Galloping Gertie."

Failure

Strong winds caused the bridge to collapse on November 7, 1940. Initially, 35 mile per hour winds excited the bridge's transverse vibration mode, with an amplitude of 1.5 feet. This motion lasted 3 hours.

The wind then increased to 42 miles per hour. In addition, a support cable at mid-span snapped, resulting in an unbalanced loading condition. The bridge response thus changed to a 0.2 Hz torsional vibration mode, with an amplitude up to 28 feet. The torsional mode is shown in Figures 1a and 1b.
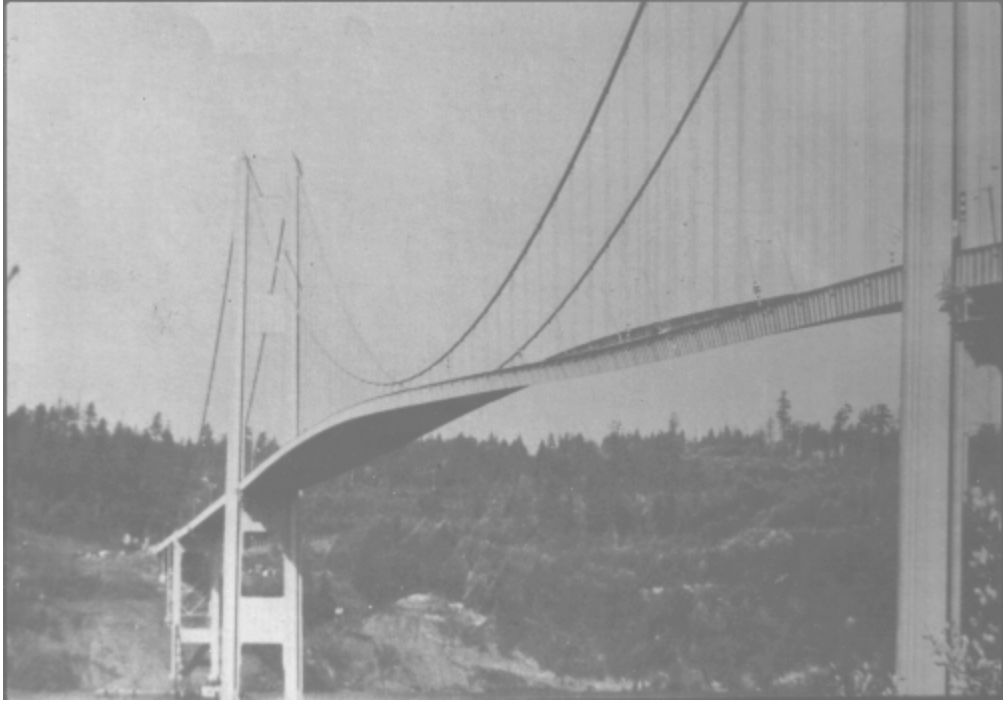
Figure 1a. Torsional Mode of the Tacoma Narrows Bridge

Figure 1b. Torsional Mode of the Tacoma Narrows Bridge

The torsional mode shape was such that the bridge was effectively divided into two halves. The two halves vibrated out-of-phase with one another. In other words, one half rotated clockwise, while the other rotated counter-clockwise. The two half spans then alternate polarities.

One explanation of this is the "law of minimum energy." A suspension bridge may either twist as a whole or divide into half spans with opposite rotations. Nature prefers the two half-span option since this requires less wind energy.

The dividing line between the two half spans is called the "nodal line." Ideally, no rotation occurs along this line.

The bridge collapsed during the excitation of this torsional mode. Specifically, a 600 foot length of the center span broke loose from the suspenders and fell a distance of 190 feet into the cold waters below. The failure is shown in Figures 2a and 2b.

Figure 2a. Failure of the Tacoma Narrows Bridge

Figure 2b. Tacoma Narrows Bridge after the Failure

<u>Failure Theories</u>

Candidates

The fundamental weakness of the Tacoma Narrows Bridge was its extreme flexibility, both vertically and in torsion.   This weakness was due to the shallowness of the stiffening girders and the narrowness of the roadway, relative to its span length.

Engineers still debate the exact cause of its collapse, however. Three theories are:

1. Random turbulence
2. Periodic vortex shedding
3. Aerodynamic instability (negative damping)

These theories are taken from Reference 1. Aerodynamic instability is the leading candidate.

Random Turbulence

An early theory was that the wind pressure simply excited the natural frequencies of the bridge. This condition is called "resonance." The problem with this theory is that resonance is a very precise phenomenon, requiring the driving force frequency to be at, or near, one of the system's natural frequencies in order to produce large oscillations. The turbulent wind pressure, however, would have varied randomly with time. Thus, turbulence would seem unlikely to have driven the observed steady oscillation of the bridge.

Vortex Shedding

Theodore von Karman, a famous aeronautical engineer, was convinced that vortex shedding drove the bridge oscillations. A diagram of vortex shedding around a spherical body is shown in Figure 3. Von Karman showed that blunt bodies such as bridge decks could also shed periodic vortices in their wakes.

A problem with this theory is that the natural vortex shedding frequency was calculated to be 1 Hz. This frequency is also called the "Strouhal frequency." The torsional mode frequency, however, was 0.2 Hz. This frequency was observed by Professor F. B. Farquharson, who witnessed the collapse of the bridge. The calculated vortex shedding frequency was five times higher than the torsional frequency. It was thus too high to have excited the torsional mode frequency. Further information is given in Appendix A.

In addition to "von Karman" vortex shedding, a flutter-like pattern of vortices may have formed at a frequency coincident with the torsional oscillation mode. Whether these flutter vortices were a cause or an effect of the twisting motion is unclear.

Figure 3. Vortex Shedding around a Spherical Body

Aerodynamic Instability

Aerodynamic instability is a self-excited vibration. In this case, the alternating force that sustains the motion is created or controlled by the motion itself. The alternating force disappears

when the motion disappears.   This phenomenon is also modeled as free vibration with negative damping.

Airfoil flutter and transmission line galloping are related examples of this instability.   Further explanations of instability are given in References 2, 3, and 4.

The following scenario shows how aerodynamic instability may have caused the Tacoma Narrows Bridge to fail. For simplicity, consider the motion of only one span half.

Assume that the wind direction was not perfectly horizontal, perhaps striking the bridge span from below, as shown in Figure 4a
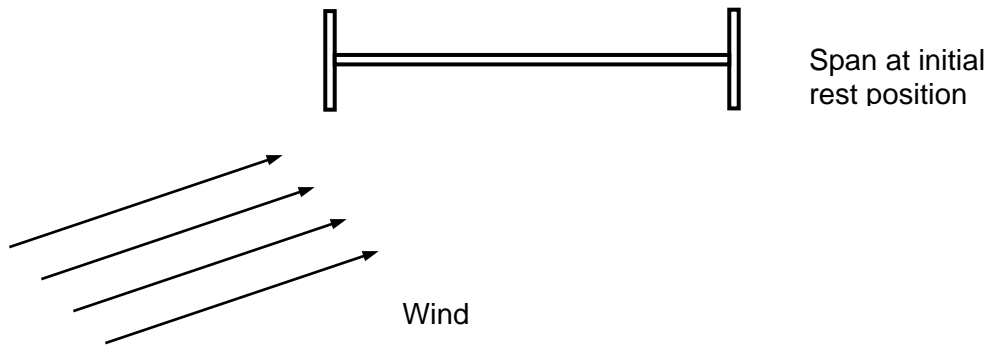

Span at initial
rest position

Wind

Figure 4a.

Thus, the bridge is initially at an angle-of-attack with respect to the wind. Aerodynamic lift is generated because the pressure below the span is greater than the pressure above. This lift force effectively places a torque, or moment, on the bridge. The span then begins to twist clockwise as show in Figure 4b. Specifically, the windward edge rotates upward while the leeward edge rotates downward.
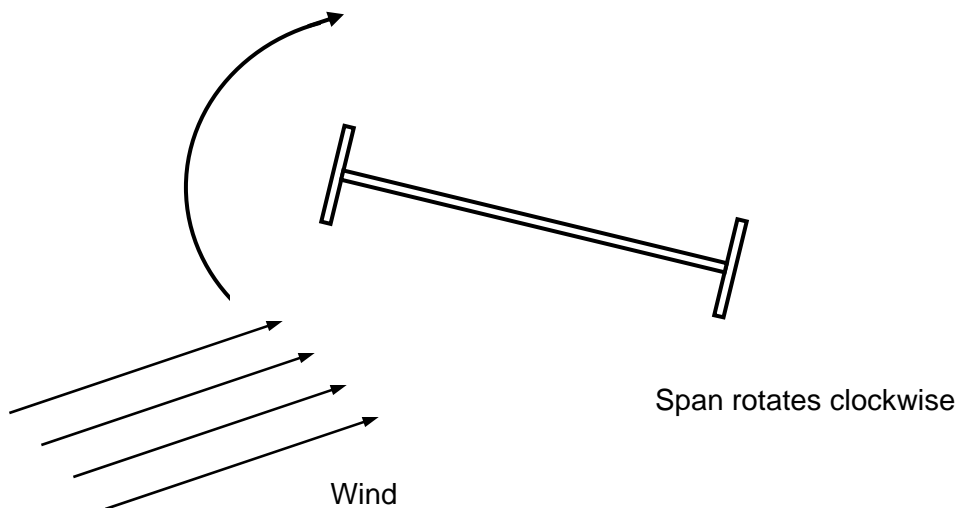

Span rotates clockwise

Wind

Figure 4b.

The span has rotational stiffness, however. Thus, elastic strain energy builds up as the span rotates. Eventually, the stiffness moment overcomes the moment from the lift force. The span then reverses its course, now rotating counter-clockwise

The span's angular momentum will not allow it to simply return to its initial rest position, however. The reason is that there is little or no energy dissipation mechanism. Thus, the span overshoots its initial rest position. In fact, it overshoots to the extent that the wind now strikes the span from above as shown in Figure 4c. The wind's lift force now effectively places a counter-clockwise moment on the span.

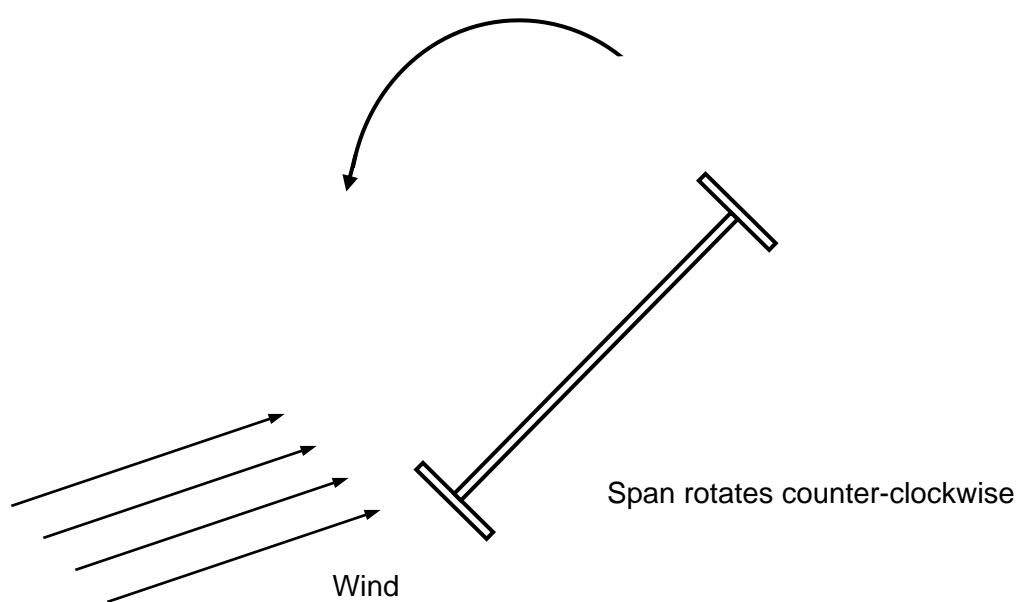Span rotates counter-clockwise

Wind

Figure 4c.

Once again, strain energy builds up in the span material. Eventually, the stiffness moment exceeds the moment from the wind's lift force. The span thus reverse course, now rotating clockwise. Again, it overshoots its rest position. The cycle of oscillation begins anew from the position shown in Figure 4a, except that the span now has rotational velocity as it passes through the original rest position.

The cycles of oscillation continue in a repetitive manner.

Note that the wind force varies as a function of the span angle during the cycle. The wind force may also vary with the angular velocity. The wind force is not a function of time, however.

Eventually, one of two failure modes occurs. One possibility is that the span experiences fatigue failure due to an excessive number of stress reversals. The other is that the angular displacement increased in an unstable manner until the material is stressed beyond its yield point, and then beyond its ultimate stress limit.

In reality, these two failure modes are interrelated. For example, accumulated fatigue effectively lowers the yield and ultimate stress limits. Regardless, the bridge collapses.

As a final note, the aerodynamic instability oscillation is not a resonant oscillation since the wind does not have a forcing frequency at, or near, the bridge's torsional mode frequency. Some physics and engineering textbooks mistakenly cite the Tacoma Narrows Bridge as an example of resonance. This problem is discussed in Reference 5.

Nevertheless, the bridge's collapse remains the most well-know structural failure due to vibration.

Replacement Bridge

A new Tacoma Narrows Bridge was built in 1950, as shown in Figure 5. The second bridge had truss-girders which allowed the winds to pass through. It also had increased torsional stiffness because it was thicker and wider. Furthermore, wind tunnel testing was performed to verify the design of the new bridge prior to its construction.

References

1. James Koughan, "The Collapse of the Tacoma Narrows Bridge, Evaluation of Competing Theories of its Demise, and the Effects of the Disaster of Succeeding Bridge Designs," The University of Texas at Austin, 1996.
2. Den Hartog, Mechanical Vibrations, McGraw-Hill, New York, 1940.
3. H. Bachmann, et al., Vibration Problems in Structures, Birkhauser Verlag, Berlin, 1995.
4. M. Levy and M. Salvadori, Why Buildings Fall Down, Norton, New York, 1992.
5. K. Billah and R. Scanlan, "Resonance, Tacoma Narrows Bridge Failure, and Undergraduate Physics, Textbooks;" American Journal of Physics, 1991.

Figure 5. The Replacement Tacoma Narrows Bridge, Built in 1950

APPENDIX A

Variables

D = diameter

f_S = Strouhal frequency

S = Strouhal number

U = Free stream velocity

Tacoma Narrows Bridge, Strouhal Calculation

The original Tacoma Narrows Bridge collapsed in 1940. It experienced severe torsional oscillations driven by a 42 mile per hour wind.

The fundamental weakness of the Tacoma Narrows Bridge was its extreme flexibility, both vertically and in torsion. This weakness was due to the shallowness of the stiffening girders and the narrowness of the roadway, relative to its span length.

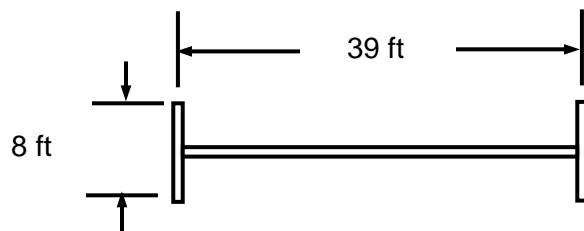A cross-section of the bridge is shown in Figure A-1.

Figure A-1. Cross-section of Tacoma Narrows Bridge Span

The Strouhal number for the bridge cross-section is S = 0.11 according to Reference 5. Furthermore, the characteristic dimension is the girder height 8 ft. The Strouhal frequency for a 42 mph (61.6 ft/sec) wind is thus

$$f_S = S \, U/D \tag{A-1}$$

$$f_S = (0.11) \ (61.1 \ \text{ft/sec}) \ / \ (\ 8 \ \text{ft}) \tag{A-2}$$

$$f_S = 0.84 \ \text{Hz} \tag{A-3}$$

The Strouhal frequency is rounded to 1 Hz in Reference 5. On the other hand, the bridge's torsional oscillation was observed to be 0.2 Hz. The Strouhal frequency was thus at least two octaves greater than the torsional mode frequency.

Billah and Scanlan thus argue strongly in Reference 5 that the Karman vortex shedding could not have driven the torsional mode oscillation. They argue instead that an aerodynamic instability resulted in a self-excited oscillation, which caused the failure. Nearly seventy years after the failure, the matter is not completely settled.