

Gauge-Invariant Quantum Gravity Corrections to Gauge Couplings via Vilkovisky-DeWitt Method and Gravity Assisted Gauge Unification

HONG-JIAN HE^{1,2}, XU-FENG WANG^{1,3}, ZHONG-ZHI XIANYU¹

¹ Center for High Energy Physics and Institute of Modern Physics, Tsinghua University, Beijing 100084, China

² Kavli Institute for Theoretical Physics China, Chinese Academy of Sciences, Beijing 100190, China

³ Institute for the Physics and Mathematics of the Universe, University of Tokyo, Chiba 277-8568, Japan

Gravity is the weakest force in nature, and the gravitational interactions with all standard model (SM) particles can be well described by perturbative expansions of the Einstein-Hilbert action as an effective theory, all the way up to energies below the fundamental Planck scale. We use Vilkovisky-DeWitt method to derive the first gauge-invariant nonzero gravitational power-law corrections to the running of gauge couplings, which make both Abel and non-Abel gauge interactions asymptotically free. We further demonstrate that the graviton-induced universal power-law runnings always assist the three SM gauge forces to reach unification around the Planck scale, irrespective of the detail of logarithmic corrections. We also compute the power-law corrections to the SM Higgs sector and derive modified triviality bound on the Higgs boson mass.

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1. INTRODUCTION

Although gravity, as the weakest force in nature, is more perturbative than the other three fundamental forces all the way up to energies below the Planck scale, it was found to be non-renormalizable in the conventional sense [1]. But this does not prevent the enormous range of successful physical and astrophysical applications of the Einstein general relativity of gravitation. In fact, all nature's four fundamental forces can be well described by the modern formulation of *effective field theories* [2], with no exception to gravitation [3]. The leading terms in the Einstein-Hilbert action,

$$S_{\text{EH}} = \int d^4x \sqrt{-g} \kappa^{-2} (R - 2\Lambda_0), \quad (1)$$

are just the least suppressed operators in the effective theory of general relativity under perturbative low energy expansion, where $\kappa^2 \equiv 16\pi G \equiv 16\pi/M_P^2$ is fixed by the Newton constant G (or Planck mass $M_P \simeq 1.2 \times 10^{19}$ GeV) and Λ_0 denotes the cosmological constant.

All standard model (SM) particles must join gravitational interaction with their couplings controlled by the universal Newton constant G . It is thus important to understand, under the effective theory formulation, how gravity corrects the SM observables, in connection to the other three gauge forces in nature. Robinson and Wilczek [4] initiated a very interesting study of gravitational corrections to running gauge couplings, but it was then realized that their calculation by using conventional background field method (BFM) [5] is generally gauge-dependent and the net result vanishes [6, 7].

However, it is important to note that Vilkovisky and DeWitt [8] proposed a new approach over the conventional BFM, especially powerful for analyses involving gravitation, which is guaranteed to be gauge-invariant, independent of the choices of both gauge-condition and

gauge-parameter [9, 10]. The Vilkovisky-DeWitt method was recently applied by Toms to study logarithmic corrections of graviton to the running coupling of QED with a nonzero cosmological constant [11] under dimensional regularization, and to scalar mass [12].

The purpose of the present work is to apply the fully gauge-invariant Vilkovisky-DeWitt method [8] for studying the gravitational corrections to the power-law running of Abel and non-Abel gauge couplings. We derive *the first gauge-invariant nonzero power-law correction, which is asymptotically free*, in support of what Robinson and Wilczek hoped. We also extend this approach for studying power-law corrections to the SM Higgs sector and derive modified triviality bound on the Higgs boson mass [13], as will be summarized in the last part of this paper. The power-law running originates from the quadratical divergences associated with graviton loop with overall couplings proportional to κ^2 . The gravitational coupling κ^2 has negative mass-dimension equal -2 , so the graviton induced loop contributions can generate generic dimensionless power-law corrections to a given gauge coupling g_i , of the form $g_i \kappa^2 \Lambda^2$, where Λ is the ultraviolet (UV) momentum cutoff. After renormalization one can deduce the generic form of one-loop Callan-Symanzik β function by general dimensional analysis,

$$\beta(g_i, \mu) = -\frac{b_{0i}}{(4\pi)^2} g_i^3 + \frac{a_0}{(4\pi)^2} (\kappa^2 \mu^2) g_i, \quad (2)$$

where μ is the renormalization scale and the coefficient a_0 has to be determined by explicit, gauge-invariant computation of graviton radiative corrections. There is no reason *a priori* to expect a_0 be exactly zero (as stressed by Robinson and Wilczek). Our key point here is to extract the physical power-law corrections via a fully gauge-invariant method à la Vilkovisky-DeWitt [8]. The physical meaning of the quadratical divergences in non-

renormalizable theories was clarified in depth by Veltman [14] and he advocated to use dimensional reduction (DRED) method [15] (rather than dimensional regularization (DREG)) to consistently regularize quadratical divergences as $d = 2$ poles for the Higgs mass corrections in the SM. Then, Einhorn and Jones made further insight [16] that any regularization procedure which preserves the right number of spin degrees of freedom for each field should give the correct results of quadratical divergence. This includes DRED but excludes DREG, as DREG is inconsistent for dealing with quadratical divergences [14, 16]. There is no need to explicitly regularize the quadratically divergent integrals until after we finish computing and summing up all their coefficients via gauge-invariant formulation, then a common momentum cutoff can be placed on the remaining single divergent integral and renormalization will be carried out to extract the power-law corrections. (This procedure was applied to extract the gauge-invariant quadratical divergence in the Higgs boson mass and was proven to be *regularization-independent* [16].) We have also explicitly used DRED method for our analysis (à la Veltman [14]) to serve as consistency check.

2. GAUGE-INVARIANT VILKOVISKY-DEWITT EFFECTIVE ACTION

The Vilkovisky-DeWitt approach [8] modifies the conventional BFM in order to build a manifestly gauge invariant effective action. The key observation is that a change in the gauge fixing condition for a gauge theory is equivalent to a change of external source term, or a field-reparametrization $\varphi_i \rightarrow \varphi'_i$ (where we use the condensed notation of DeWitt [17], with the subscript i denoting all internal and spacetime indices besides the spacetime coordinates). Note that the classical action $S[\varphi]$ is reparametrization invariant and thus behaves as a “scalar” under field reparametrization; but the effective action $\Gamma[\bar{\varphi}]$ in the conventional BFM is not a scalar. This fact can be understood from the definition of conventional $\Gamma[\bar{\varphi}]$,

$$e^{i\Gamma[\bar{\varphi}]} = \int d\varphi \mu[\varphi] \exp i \left[S[\varphi] + (\bar{\varphi}^i - \varphi^i) \frac{\delta\Gamma[\bar{\varphi}]}{\delta\bar{\varphi}^i} \right], \quad (3)$$

where $\bar{\varphi}^i$ denotes the background of φ^i , and $d\varphi \mu[\varphi]$ is the measure of functional integral. For a gauge theory, $\mu[\varphi]$ contains gauge-fixing condition and the corresponding DeWitt-Faddeev-Popov determinant. The last term in the exponent is generally not a scalar under the field reparametrization. This fact can be easily understood if we take geometrical viewpoint and treat the field configuration space as a manifold [8]. Then, it is clear that the difference of two distinct points $\bar{\varphi}^i - \varphi^i$ is generally not well defined, in the sense that it is not field-reparametrization covariant. As a result, the effective

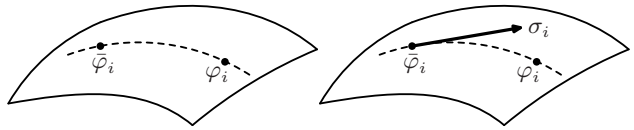


FIG. 1: Coordinates difference in field space as a manifold.

action $\Gamma[\bar{\varphi}]$ may depend on the choice of gauge condition.

The way out of this trouble is to replace the coordinate difference $\bar{\varphi}^i - \varphi^i$ by a covariant vector $\sigma^i[\varphi, \bar{\varphi}]$ as illustrated in Fig. 1, and introduce a connection Γ_{jk}^i in field space (with which the parallel transport can be defined). Thus, the Vilkovisky-DeWitt effective action Γ_G , as a scalar in the field space, can be constructed as [8],

$$e^{i\Gamma_G[\bar{\varphi}]} = \int d\varphi \mu[\varphi] \exp i \left[S[\varphi] + C_j^{-1i}[\bar{\varphi}] \Gamma_{G,i}[\bar{\varphi}] \sigma^j[\bar{\varphi}, \varphi] \right], \quad (4)$$

where $\Gamma_{G,i} \equiv \delta\Gamma_G/\delta\bar{\varphi}^i$ and we will always use a subscript “comma” to denote the functional derivative. Here σ^i is the vector introduced in field space, and with the aid of connection Γ_{jk}^i , can be expanded as

$$\sigma^i[\varphi, \varphi'] = \varphi^i - \varphi'^i - \frac{1}{2} \Gamma_{jk}^i (\varphi - \varphi')^j (\varphi - \varphi')^k + \dots \quad (5)$$

The coefficient C_j^{-1i} can also be expanded perturbatively,

$$C_j^{-1i}[\bar{\varphi}, \varphi] = \delta_j^i + \frac{1}{3} R_{mni}^j[\bar{\varphi}] \sigma^m[\bar{\varphi}, \varphi] \sigma^n[\bar{\varphi}, \varphi] + \dots, \quad (6)$$

where R_{mni}^j is the curvature tensor associated with connection Γ_{jk}^i .

Under perturbative expansion, we can write down one-loop Vilkovisky-DeWitt effective action, which is also a scalar under reparametrization,

$$\Gamma_G[\bar{\varphi}] = S[\bar{\varphi}] - i \ln \mu[\bar{\varphi}] + \frac{i}{2} \text{Tr} \ln \nabla_m \nabla_n S, \quad (7)$$

where ∇_m is the covariant derivative associated with connection Γ_{mn}^i ,

$$\frac{\delta^2 S}{\delta\varphi^m \delta\varphi^n} \rightarrow \nabla_m \nabla_n S = \frac{\delta^2 S}{\delta\varphi^m \delta\varphi^n} - \Gamma_{mn}^i(\varphi) \frac{\delta S}{\delta\varphi^i}. \quad (8)$$

For a gauge theory, consider the infinitesimal gauge transformation,

$$\delta\varphi^i = K_\alpha^i[\varphi] \epsilon^\alpha, \quad (9)$$

with $K_\alpha^i[\varphi]$ being the generators of gauge transformation and ϵ^α the infinitesimal gauge parameters. Thus, for quantization the gauge-fixing condition $\chi_\alpha(\varphi)$ and the DeWitt-Faddeev-Popov ghost term $Q_{\alpha\beta}[\bar{\varphi}] = \frac{\delta\chi_\alpha}{\delta\epsilon^\beta}$ should be introduced. So, the Vilkovisky-DeWitt effective action is given by, up to one-loop order,

$$\Gamma[\bar{\varphi}] = S[\bar{\varphi}] - \ln \det Q_{\alpha\beta}[\bar{\varphi}]$$

$$+\frac{i}{2}\text{Tr}\ln\left(\nabla_m\nabla_n S+\frac{1}{2\xi}(\chi_\alpha\chi^\alpha)_{,\hat{\varphi},\hat{\varphi}}[\hat{\varphi}]\right), \quad (10)$$

which is proven to be invariant under the change of gauge condition χ_α and gauge-fixing parameter ξ [8, 9]. We have defined the background field $\bar{\varphi}^i$ and fluctuating field $\hat{\varphi}^i$ via $\varphi^i = \bar{\varphi}^i + \hat{\varphi}^i$. For calculations in a specific gauge theory, the connection Γ_{jk}^i is very complicated and non-local. But it can be shown that, when the Landau-DeWitt gauge condition

$$\chi_\alpha[\bar{\varphi}, \hat{\varphi}] = K_{\alpha i}[\bar{\varphi}]\hat{\varphi}^i = 0 \quad (11)$$

is chosen, the relevant parts of connection coefficients are simply given by the Christoffel symbol associated with a metric G_{ij} in field space [18],

$$\Gamma_{jk}^i = \frac{1}{2}G^{i\ell}(G_{\ell j,k} + G_{\ell k,j} - G_{jk,\ell}). \quad (12)$$

In summary, Vilkovisky-DeWitt effective action provides a fully gauge-invariant description of the off-shell gauge field theories, which is guaranteed to be independent of choices of both gauge-condition and gauge-parameter. In the following, we will apply this method to analyze the quantum gravity coupled to the Abel and non-Abel gauge theories, as well as the SM Higgs sector.

3. GRAVITATIONAL CORRECTIONS TO ABEL AND NON-ABEL β FUNCTIONS

We start from the classical action of Einstein-Maxwell theory, which consists of the Einstein-Hilbert action (1) and

$$S_{\text{EM}} = -\frac{1}{4}\int d^4x\sqrt{-g}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}. \quad (13)$$

Since the Vilkovisky-DeWitt method does not require on-shell background, we expand the metric $g_{\mu\nu}$ around the Minkowski background $\eta_{\mu\nu}$,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \quad (14)$$

We further split the gauge field A_μ as

$$A_\mu = \bar{A}_\mu + a_\mu, \quad (15)$$

with \bar{A}_μ the background field and a_μ the quantum fluctuating field. As shown in Eq. (12), the connection Γ_{jk}^i is determined from the metric G_{ij} defined in the field manifold. There is a natural choice [8] of the field-space metric G_{ij} with its nonzero components given by

$$G_{g_{\mu\nu}(x)g_{\alpha\beta}(y)} = \frac{\sqrt{-g}}{2\kappa^2}(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - g^{\mu\nu}g^{\alpha\beta})\delta(x-y),$$

$$G_{A_\mu(x)A_\nu(y)} = \sqrt{-g}g^{\mu\nu}\delta(x-y). \quad (16)$$

Thus, the relevant part of connection can be derived from Eq. (12) under the Landau-DeWitt gauge condition (11),

$$\Gamma_{g_{\mu\nu}(y)g_{\alpha\beta}(z)}^{g_{\rho\sigma}(x)} = \frac{1}{4}g^{\mu\nu}\delta_\rho^{(\alpha}\delta_\sigma^{\beta)} + \frac{1}{4}g^{\alpha\beta}\delta_\rho^{(\mu}\delta_\sigma^{\nu)} - \delta_{(\rho}^{(\alpha}g^{\beta)(\mu}\delta_\sigma^{\nu)} + \frac{1}{4(d-2)}g_{\rho\sigma}(2g^{\mu(\alpha}g^{\beta)\nu} - g^{\mu\nu}g^{\alpha\beta}) \times \delta(x-y)\delta(x-z),$$

$$\Gamma_{A_\alpha(y)A_\beta(z)}^{g_{\mu\nu}(x)} = \frac{1}{4}\kappa^2\delta_{(\mu}^\alpha\delta_{\nu)}^\beta\delta(x-y)\delta(x-z), \quad (17)$$

$$\Gamma_{A_\nu(y)g_{\alpha\beta}(z)}^{A_\mu(x)} = \frac{1}{4}(\delta_\mu^\nu g^{\alpha\beta} - \delta_\mu^\alpha g^{\nu\beta} - \delta_\mu^\beta g^{\nu\alpha})\delta(x-y)\delta(x-z),$$

where we have introduced the symmetrization notation, $\delta_\rho^{(\alpha}\delta_\sigma^{\beta)} = \frac{1}{2}(\delta_\rho^\alpha\delta_\sigma^\beta + \delta_\rho^\beta\delta_\sigma^\alpha)$, and so on. The Landau-DeWitt gauge condition should be determined by the gauge transformations,

$$\delta g_{\mu\nu} = -g_{\mu\alpha}\partial_\nu\epsilon^\alpha - g_{\nu\alpha}\partial_\mu\epsilon^\alpha - \epsilon^\alpha\partial_\alpha g_{\mu\nu}, \quad (18a)$$

$$\delta A_\mu = -\partial_\mu\epsilon - A_\nu\partial_\mu\epsilon^\nu - \epsilon^\nu\partial_\nu A_\mu, \quad (18b)$$

with ϵ^α being the infinitesimal parameter of gravitational gauge transformation and ϵ the infinitesimal parameter of $U(1)$ gauge transformation. Then, the gauge-fixing functions for photon and graviton fields are

$$\chi = \partial^\mu a_\mu, \quad (19a)$$

$$\chi_\mu = \left(\partial^\lambda h_{\mu\lambda} - \frac{1}{2}\partial_\mu h\right) + \frac{\kappa}{2}a^\lambda\bar{F}_{\mu\lambda}, \quad (19b)$$

where $\bar{F}_{\mu\nu} = \partial_\mu\bar{A}_\nu - \partial_\nu\bar{A}_\mu$ and an overall factor $-2/\kappa$ is factorized out in (19b) for the convenience of normalization. So the Lagrangian contains the following gauge-fixing terms,

$$\mathcal{L}_{\text{gf}} = \frac{1}{2\xi}\chi_\mu\chi^\mu - \frac{1}{2\xi}\chi^2, \quad (20)$$

where ξ (ζ) is gauge-fixing parameter for photon (graviton) field, and will be set to zero at the end of calculation, as required by imposing the Landau-DeWitt gauge condition.

Now consider the connection-induced terms in the Lagrangian,

$$\mathcal{L}_{\text{con}} = -\frac{1}{2}\Gamma_{A_\alpha A_\beta}^{g_{\mu\nu}}S_{,g_{\mu\nu}}a_\alpha a_\beta - \frac{1}{2}\kappa^2\Gamma_{g_{\mu\nu}g_{\alpha\beta}}^{g_{\rho\sigma}}S_{,g_{\rho\sigma}}h_{\mu\nu}h_{\alpha\beta} - \kappa\Gamma_{A_\nu g_{\alpha\beta}}^{A_\mu}S_{,A_\mu}a_\nu h_{\alpha\beta}, \quad (21)$$

where

$$S_{,g_{\mu\nu}}|_{\bar{\varphi}} = -\frac{\Lambda_0}{\kappa^2}\eta^{\mu\nu} - \frac{1}{8}\eta^{\mu\nu}\bar{F}_{\alpha\beta}\bar{F}^{\alpha\beta} + \frac{1}{2}\bar{F}_\lambda^\mu\bar{F}^{\nu\lambda}, \quad (22a)$$

$$S_{,A_\mu}|_{\bar{\varphi}} = \partial_\nu\bar{F}^{\nu\mu}. \quad (22b)$$

The ghost part of the Lagrangian is given by

$$\mathcal{L}_{\text{gh}} = \bar{\eta}^\lambda\delta\chi_\lambda + \bar{\eta}\delta\chi, \quad (23)$$

where $\delta\chi$ and $\delta\chi_\lambda$ are the changes under gauge transformations with $\epsilon = \eta$ and $\epsilon^\lambda = \eta^\lambda$, and (η, η^λ) the anti-commuting ghost fields.

With these we can sum up all required terms for computing the effective action (10),

$$S_Q = S_{\text{EM}} + \int d^4x (\mathcal{L}_{\text{con}} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}}). \quad (24)$$

The one-loop effective potential will be deduced from $\frac{i}{2}\text{Tr}\ln(S_Q)_{,i,j}$, according to Eq.(10). This has clear diagrammatic interpretation. For our purpose we are interested in all the bilinear terms of background photon field, which correspond to the one-loop self-energy diagrams listed in Fig. 2.

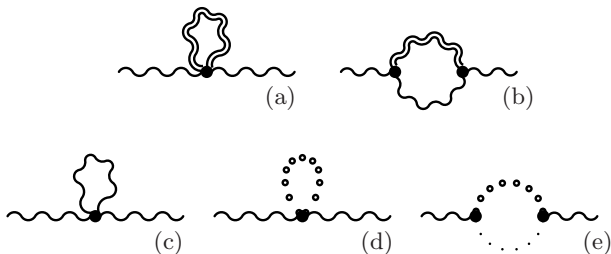


FIG. 2: Graviton induced radiative corrections for photon self-energy: the wavy external (internal) lines stand for background (fluctuating) photon fields, the double-lines for gravitons, the dotted lines for photon-ghosts, and the circle-dotted line for graviton ghosts.

We note that among all five diagrams in Fig. 2(a)-(e), the first two also exist in the conventional BFM or diagrammatic calculation (though the present couplings differ from the conventional ones), but the last three arise solely from the *connection-induced contributions* in the Vilkovisky-DeWitt formulation. We systematically compute these diagrams using the Feynman rules from Eq. (24), and only keep the quadratic divergent parts of the loop integrals,

$$(a) = \frac{3(1+\zeta)}{2}\kappa^2(p^2g_{\mu\nu} - p_\mu p_\nu) \mathcal{I}_2, \quad (25a)$$

$$(b) = \left(-\frac{3+\xi}{8\zeta} - \frac{3}{2}\zeta\right)\kappa^2(p^2g_{\mu\nu} - p_\mu p_\nu) \mathcal{I}_2, \quad (25b)$$

$$(c) = \frac{3+\xi}{8\zeta}\kappa^2(p^2g_{\mu\nu} - p_\mu p_\nu) \mathcal{I}_2, \quad (25c)$$

$$(d) = -\kappa^2(p^2g_{\mu\nu} - p_\mu p_\nu) \mathcal{I}_2, \quad (25d)$$

$$(e) = -\frac{1}{4}\kappa^2(p^2g_{\mu\nu} - p_\mu p_\nu) \mathcal{I}_2, \quad (25e)$$

where the integral

$$\mathcal{I}_2 \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \quad (26)$$

is quadratically divergent by power-counting, though we need not to specify a regularization of it for the calcula-

tions so far [16]. Summing up all the self-energy contributions (25a)-(25e), we find that all $\frac{1}{\zeta}$ poles explicitly cancel, which is a consistency check for Vilkovisky-DeWitt method, and we deduce the net result in the Landau-DeWitt gauge,

$$(a)+(b)+(c)+(d)+(e) = \frac{\kappa^2}{4}(p^2g_{\mu\nu} - p_\mu p_\nu) \mathcal{I}_2. \quad (27)$$

As guaranteed by the Vilkovisky-DeWitt method [8, 9], this is a fully gauge-invariant result, and now we are free to regularize the quadratically divergent integral (26) by placing a physical momentum cutoff Λ ,

$$\mathcal{I}_2 = -i\frac{\Lambda^2}{16\pi^2}. \quad (28)$$

Thus, we can deduce the QED gauge-coupling renormalization,

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} - \frac{\kappa^2(\Lambda^2 - \mu^2)}{64\pi^2 g^2}, \quad (29)$$

under the minimal subtraction scheme, and the corresponding renormalization constant,

$$Z_g = 1 - \frac{\kappa^2(\Lambda^2 - \mu^2)}{128\pi^2}, \quad (30)$$

where $g(\Lambda) = Z_g g(\mu)$ and μ is the renormalization scale.

From Eq. (29) or Eq. (30), we finally derive the gauge-invariant gravitational *power-law correction* to the QED β -function,

$$\Delta\beta(g, \mu) = -\frac{1}{64\pi^2}(\kappa^2\mu^2)g, \quad (31)$$

which is *asymptotically free* and gives $a_0 = -\frac{1}{4}$ in Eq. (2). There is no logarithmic graviton correction to the gauge coupling β -function in the absence of cosmological constant [7], since dimensional counting shows that gravitational logarithmic corrections could only contribute to dimension-6 operators such as $(D_\mu F^{\mu\nu})^2$ rather than the standard dimension-4 gauge kinetic term (13). With a nonzero cosmological constant Λ_0 in (1), the graviton-induced logarithmic correction can appear [11] since Λ_0 has mass-dimension equal 2 and thus the product $\Lambda_0\kappa^2$ provides a proper dimensionless parameter for one-loop gravitational logarithmic correction. We stress that due to the nonzero new result in the above Eq.(31), *the graviton-induced leading power-law correction will eventually dominate gauge coupling running at high scales and always drive gauge unification nearby the Planck scale, irrespective of the detail of all logarithmic corrections.* This will be demonstrated in the next section.

As a consistency check, we have also explicitly used the DRED method to regularize the quadratical divergence as $d = 2$ poles (à la Veltman [14]) and then identify these poles as quadratical divergences at $d = 4$;

the outcome is the same as our Eqs.(25)-(27), which confirms the insight of Einhorn and Jones [16] that the gauge-invariant quadratical divergence can be extracted in a regularization-independent way, so long as the spin-degrees of freedom for the external fields are correctly preserved.

For comparison, we want to clarify how our above analysis differs from that of the conventional BFM (or the equivalent diagrammatical) approach. The latter corresponds to setting all the Γ_{jk}^i related connection terms in Sec. 2-3 vanish. In consequence, only the diagrams (a)-(b) in Fig. 2 survive, and we find that now they exactly cancel with each other,

$$\begin{aligned} \text{(a)} &= -\text{(b)} = \frac{3(1+\zeta)}{2}\kappa^2(p^2 g_{\mu\nu} - p_\mu p_\nu)\mathcal{I}_2, \\ \text{(a)} + \text{(b)} &= 0. \end{aligned} \quad (32)$$

This also agrees to the null result of the conventional diagrammatic calculation in the second paper of [7]. Hence, we have further confirmed that our nonzero power-law correction in Eq. (31) solely arises from the connection-induced new contributions to Fig. 2(b)-(e) via the gauge-invariant Vilkovisky-DeWitt effective action; and this explains why it was not discovered before.

As one more consistency check of the present calculation, we have used an another way [11] to compute the Vilkovisky-DeWitt effective action in the coordinate space and with the aid of Wick theorem. Let us define the photon and graviton propagators,

$$\langle a_\mu(x)a_\nu(y) \rangle = D_{\mu\nu}(x, y), \quad (33a)$$

$$\langle h_{\mu\nu}(x)h_{\alpha\beta}(y) \rangle = D_{\mu\nu,\alpha\beta}(x, y), \quad (33b)$$

with

$$D_{\mu\nu}(x, y) = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} D_{\mu\nu}(k), \quad (34a)$$

$$D_{\mu\nu,\alpha\beta}(x, y) = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} D_{\mu\nu,\alpha\beta}(k). \quad (34b)$$

Thus, we derive the effective action for gauge field,

$$i\Gamma_A = \langle iS_2 \rangle - \frac{1}{2}\langle S_1^2 \rangle \quad (35)$$

where S_1 and S_2 are the action terms containing one and two external fields, respectively. Then we compute the effective action using the CADABRA package [19], and deduce the gauge-part,

$$i\Gamma_A = \frac{3\kappa^2}{2}\mathcal{I}_2 \int d^4x \frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu}, \quad (36)$$

corresponding to diagrams in Fig. 2(a)-(c), and the sum of Eqs. (25a)-(25c). For ghost-part, we derive

$$i\Gamma_{\text{gh}} = \langle iS_{2\text{gh}} \rangle - \frac{1}{2}\langle S_{1\text{gh}}^2 \rangle$$

$$= -\frac{5\kappa^2}{4}\mathcal{I}_2 \int d^4x \frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu}, \quad (37)$$

which corresponds to diagrams in Fig. 2(d)-(e), and the sum of Eqs. (25d)-(25e). With these we obtain the full one-loop effective action,

$$\Gamma = \Gamma_A + \Gamma_{\text{gh}} = -\frac{\kappa^2\Lambda^2}{64\pi^2} \int d^4x \frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu}, \quad (38)$$

from which we reproduce the same gauge coupling renormalization as in (29)-(30) and the same power-law correction to the β function as in (31).

Next, we discuss the extension of the above analysis to non-Abelian gauge theories coupled to Einstein gravity. We first note that a non-Abelian gauge theory adds no more graviton-induced self-energy diagram beyond those given in Fig. 2, except the couplings in these diagrams may differ from QED. So, let us inspect the possible change in each diagram of Fig. 2 for the non-Abelian case.

First, Figs. 2(a) and 2(d) contain pure gravitational interactions only, so they remain the same for non-Abelian theories. Second, Figs. 2(b) and 2(e) do not change too, since both graviton and graviton-ghost carry no gauge-charge. So both the gauge fields in and outside the loop must share the same ‘‘color’’ and thus no extra summation of ‘‘color’’ over the loop gauge-field. Third, Fig. 2(c) could receive a change due to possible ‘‘color’’ summation over the gauge-loop. The relevant changes would come from two places at one-loop level. One is the gauge-fixing (19b), which contributes a term of the following form,

$$\bar{F}_{\mu\nu}^a \bar{F}_{\rho\sigma}^b a_\alpha^a a_\beta^b. \quad (39)$$

Thus, given the two external background gauge-fields (from $\bar{F}_{\mu\nu}^a \bar{F}_{\rho\sigma}^b$) for Fig. 2(c), there is no more summation over the gauge-indices of the fluctuating gauge-field in the loop. The other contribution comes from the connection term, namely the first term in (21), which would contribute to Fig. 2(c) via the form,

$$\bar{F}_{\mu\nu}^a \bar{F}_{\rho\sigma}^a a_\lambda^b a_\kappa^b. \quad (40)$$

This allows a summation over the loop gauge-indices ‘‘b’’ and enhances the contribution by an overall factor of the number of non-Abelian gauge fields, which equals $N^2 - 1$ for the $SU(N)$ gauge group. But our explicit calculation shows that this connection-induced contribution actually vanishes for both Abelian and non-Abelian cases. Hence, the conclusion is that our graviton induced power-law correction (31) is universal for both Abelian and non-Abelian gauge theories.

4. GRAVITY ASSISTED GAUGE UNIFICATION

Gauge coupling unification is a beautiful idea that suggests the three apparently different gauge couplings of

the SM (as measured at low energies) would converge to a single coupling of the grand unification (GUT) group [20] at high scales. The evolutions of gauge couplings from low scale to GUT scale are conventionally governed by the renormalization group equations (RGEs) with logarithmic running [21]. But the precision data show that logarithmic evolutions of the three gauge couplings do not exactly converge for the SM particle spectrum [22], while the convergence works fine in the minimal supersymmetric standard model (MSSM) with one-loop RG running. But more precise numerical analyses including two-loop RG running reveal that even in MSSM the strong gauge coupling α_3 does not exactly meet with the other two at the GUT scale as its value is smaller than (α_1, α_2) by about 3%; so it is necessary to carefully invoke the model-dependent one-loop threshold effects [23]. It was also argued that the gravity-induced effective higher dimensional operators can generate uncertainties larger than the usual two-loop effects of MSSM and thus significantly alter the gauge unification [24].

With the gauge-invariant gravitational power-law corrections (31), we can resolve running gauge coupling $\alpha_i(\mu) = g_i^2(\mu)/4\pi$ from the RGE (2),

$$\frac{e^{-c\mu^2}}{\alpha_i(\mu)} - \frac{e^{-c\mu_0^2}}{\alpha_i(\mu_0)} = \frac{b_{0i}}{4\pi} \int_{\mu_0^2}^{\mu^2} \frac{dx}{x} e^{-cx}, \quad (41)$$

with $c \equiv \frac{|a_0|\kappa^2}{(4\pi)^2} = \frac{1}{4\pi M_P^2}$. So we further deduce,

$$\alpha_i(\mu) = \frac{\alpha_i(\mu_0)e^{-c\mu^2}}{e^{-c\mu_0^2} + \frac{b_{0i}\alpha_i(\mu_0)}{4\pi} \int_{\mu_0^2}^{\mu^2} \frac{dx}{x} e^{-cx}} \quad (42a)$$

$$\simeq \frac{\alpha_i(\mu_0) \exp\left[-\frac{\mu^2}{4\pi M_P^2}\right]}{1 + \frac{b_{0i}\alpha_i(\mu_0)}{4\pi} \left[\ln \frac{\mu^2}{\mu_0^2} + \frac{\mu^2}{4\pi M_P^2}\right]}, \quad (42b)$$

where for the estimate in (42b) we have kept in mind that $\mu_0 \lll M_P$ (such as the choice of $\mu_0 = M_Z$ below), and we also expanded the exponential integral for $\mu < \sqrt{4\pi}M_P \simeq 4.33 \times 10^{19}$ (which holds for most energy region in Fig. 3). Eq.(42b) explicitly shows that the evolution of any gauge coupling $\alpha_i(\mu)$ will be exponentially suppressed by $\exp\left[-\frac{\mu^2}{4\pi M_P^2}\right]$, which dominates the running behavior for high scales above $O(10^{-2}M_P)$. Hence, the universal gravitational power-law corrections will *always drive all gauge couplings to rapidly converge to the UV fixed point at high scales and reach unification around the Planck scale, irrespective of the detail of their logarithmic corrections and initial values*. This feature is numerically demonstrated in Fig. 3 by using the evolution equation (42a).

In Fig. 3(a)-(b), we have analyzed the gauge coupling runnings for both the SM and the MSSM, where the conventional coefficients b_{0i} in the one-loop RGEs

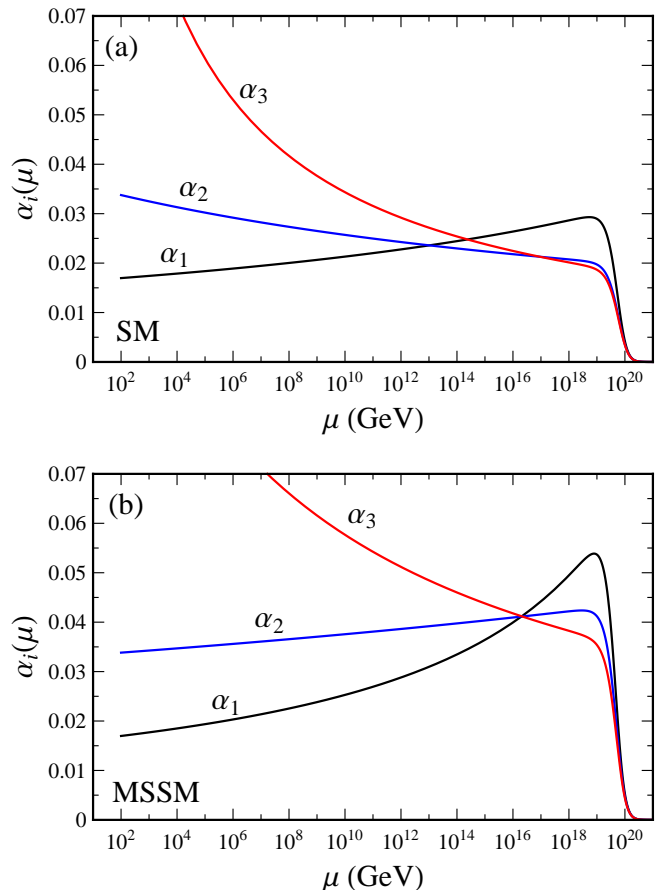


FIG. 3: Gravity assisted gauge unifications in the SM [plot-(a)] and in the MSSM [plot-(b)]. In each case the graviton-induced universal power-law runnings become dominant above 10^{18} GeV and always drive three gauge couplings to rapidly converge towards the UV fixed point, nearby the Planck scale.

are, $(b_{01}, b_{02}, b_{03}) = (-\frac{41}{10}, \frac{19}{6}, 7)$ for the SM and $(b_{01}, b_{02}, b_{03}) = (-\frac{33}{5}, -1, 3)$ for the MSSM. We have also input the initial values, $\alpha_1^{-1} = 59.00 \pm 0.02$, $\alpha_2^{-1} = 29.57 \pm 0.02$, and $\alpha_3^{-1} = 8.50 \pm 0.14$, at the Z -pole $\mu_0 = M_Z$ [22].

For the graviton-induced one-loop β -function (2) with a_0 given in (31), we note that at the Planck scale $\mu = M_P$, the loop-expansion parameter, $\frac{a_0}{(4\pi)^2}(\kappa^2\mu^2) = \frac{-1}{4\pi} \frac{\mu^2}{M_P^2} = \frac{-1}{4\pi} = O(10^{-1})$, is still fairly small, and the condition $\frac{1}{4\pi} \frac{\mu^2}{M_P^2} < 1$ holds for $\mu < \sqrt{4\pi}M_P \simeq 4.33 \times 10^{19}$ GeV, which is better than the naive expectation. Fig. 3 shows that above 10^{19} GeV the gauge couplings rapidly converge due to the exponential suppression in (42a); and quite before approaching the UV fixed point the three curves become indistinguishable for $\alpha_i \lesssim 0.01$ (SM) and $\alpha_i \lesssim 0.02$ (MSSM), corresponding to a scale of $(5 - 6) \times 10^{19}$ GeV which is slightly above $\sqrt{4\pi}M_P$ but clearly below 10^{20} GeV. For the perturbative one-

loop running of gauge couplings, the renormalization of κ^2 belongs to higher order effect here and is thus not included. One may concern the running behavior of κ^2 for scales above $\sqrt{4\pi}M_P$ and related higher order effects. It is useful to note that there are clear evidences supporting Einstein gravity to be asymptotically safe via the existence of nontrivial UV fixed point in its RG flow, so it may be UV complete and perturbatively sensible even beyond the Planck scale [25][26]. To be conservative, we would consider the one-loop prediction of Fig. 3 as an instructive extrapolation of the Einstein gravity, since this provides an encouraging insight on the important role of gravity for realizing the gauge unification, and should strongly inspire more elaborate investigations along this direction.

With these said, we make a few more comments. Fig. 3(a) shows that for the SM gauge coupling evolutions, despite their familiar non-convergence in the region of 10^{13-17} GeV, the three couplings do unify around the Planck scale. For the MSSM as shown in Fig. 3(b), one needs not to worry about the model-dependent threshold effects or the two-loop-induced non-convergence around the scale of 10^{16} GeV, it is quite possible that the GUT does not happen around the scale of 10^{16} GeV, as in the SM case. Instead, the real GUT would be naturally realized around the Planck scale, and thus is expected to simultaneously unify with the gravity force as well. This also removes the old puzzle on why the conventional GUT scale is about three orders of magnitude lower than the fundamental Planck scale. Furthermore, the Planck scale unification helps to sufficiently postpone nucleon decays, which explains why all the experimental data so far support the proton stability. In addition, this is also a good news for various approaches of dynamical electroweak symmetry breaking [27], such as the technicolor type of theories, which often invoke many gauge groups with new strong forces at the intermediate scales and makes one worry about whether a gauge unification could ever be realized at certain high scales in these theories. Fortunately, the universal gravitational power-law running for all gauge couplings found above should ensure a final unification around the Planck scale.

5. GRAVITATIONAL CORRECTION TO HIGGS BOSON COUPLING AND MASS

Without losing generality, we first consider a real scalar field which minimally couples to Einstein gravity,

$$S_\phi = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \quad (43)$$

where $V(\phi)$ is the scalar potential, $V = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$, with $m^2 \geq 0$ ($m^2 < 0$) corresponding to the unbroken (broken) phase. To compute the effective potential

of scalar field, we expand the graviton and scalar fields around their backgrounds,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad \phi = \bar{\phi} + \hat{\phi}. \quad (44)$$

We impose the Landau-DeWitt gauge condition for the gauge-fixing,

$$\mathcal{L}_{\text{gf}} = \frac{1}{2\zeta} \left(h_{\nu,\mu}^\mu - \frac{1}{2} h_{,\nu} - \frac{\kappa}{2} \hat{\phi} \partial_\nu \bar{\phi} \right)^2. \quad (45)$$

Then, we derive the connection-induced terms in the graviton-scalar sector of the Lagrangian,

$$\begin{aligned} \mathcal{L}_{\text{conn}} = & -\frac{\kappa^2}{2} S_{,g_{\mu\nu}} \Gamma_{g_{\alpha\beta}g_{\rho\sigma}}^{g_{\mu\nu}} h_{\alpha\beta} h_{\rho\sigma} - \frac{1}{2} S_{,g_{\mu\nu}} \Gamma_{\phi\phi}^{g_{\mu\nu}} \hat{\phi}^2 \\ & - \kappa S_{,\phi} \Gamma_{\phi g_{\mu\nu}}^\phi \hat{\phi} h_{\mu\nu}, \end{aligned} \quad (46)$$

with

$$S_{,g_{\mu\nu}} = -\frac{1}{2} \partial^\mu \bar{\phi} \partial^\nu \bar{\phi} + \frac{1}{2} \eta^{\mu\nu} \left[\frac{1}{2} \partial_\rho \bar{\phi} \partial^\rho \bar{\phi} - V(\bar{\phi}) \right], \quad (47a)$$

$$S_{,\phi} = -\partial^2 \bar{\phi} - m^2 \bar{\phi} - \frac{\lambda}{6} \bar{\phi}^3. \quad (47b)$$

In (46) the graviton-field connection $\Gamma_{g_{\alpha\beta}g_{\rho\sigma}}^{g_{\mu\nu}}$ was given in Eq. (17), and the scalar-field related connections are,

$$\Gamma_{\phi(y)\phi(z)}^{g_{\mu\nu}(x)} = \frac{\kappa^2}{4} g_{\mu\nu} \delta(x-y) \delta(x-z), \quad (48a)$$

$$\Gamma_{\phi(y)g_{\mu\nu}(z)}^{\phi(x)} = \frac{1}{4} g^{\mu\nu} \delta(x-y) \delta(x-z), \quad (48b)$$

$$\Gamma_{\phi(y)\phi(z)}^{\phi(x)} = \Gamma_{\phi(y)g^{\mu\nu}(z)}^{g_{\alpha\beta}(x)} = 0, \quad (48c)$$

which are derived from the metrics $G_{\phi(x)\phi(y)} = \sqrt{-g} \delta(x-y)$ and $G_{g_{\mu\nu}(x)g_{\alpha\beta}(y)}$ [as in Eq. (16)]. The Feynman diagrams for the graviton-induced scalar self-energy corrections and quartic vertex corrections are shown in Fig. 4 and Fig. 5, respectively.

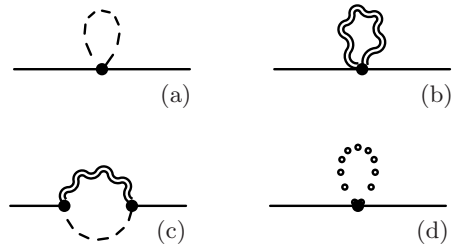


FIG. 4: Graviton-induced self-energy for scalar field: dashed-line is fluctuating scalar field, double-wavy-line denotes graviton, and circle-dotted-line depicts graviton-ghost.

For the scalar self-energy corrections, we compute the quadratic divergent part for each diagram in Fig. 4,

$$\text{Fig.4(a)} = \left[\frac{1}{2} \lambda + \kappa^2 \left(\frac{1}{8} - \frac{1}{4\zeta} \right) p^2 - \frac{1}{4} \kappa^2 m^2 \right] \mathcal{I}_2, \quad (49a)$$

Graviton-Induced Corrections	Conventional Approach	Connection-Induced only (from VDA)	Sum (Our Results) (Landau-DeWitt $\zeta \rightarrow 0$)
Fig.4(a)	0	$(\frac{1}{8} - \frac{1}{4\zeta})p^2 - \frac{1}{4}m^2$	$(\frac{1}{8} - \frac{1}{4\zeta})p^2 - \frac{1}{4}m^2$
Fig.4(b)	$-(3 + 2\zeta)m^2$	0	$-3m^2$
Fig.4(c)	ζp^2	$(-1 + \frac{1}{4\zeta})p^2$	$(-1 + \frac{1}{4\zeta})p^2$
Fig.4(d)	0	p^2	p^2
Γ_2	$\zeta p^2 - (3 + 2\zeta)m^2$	$\frac{1}{8}p^2 - \frac{1}{4}m^2$	$\frac{1}{8}(p^2 - 26m^2)$
Fig.5(a)	0	$-\frac{1}{4}\lambda$	$-\frac{1}{4}\lambda$
Fig.5(b)	$-(3 + 2\zeta)\lambda$	0	-3λ
Γ_4	$-(3 + 2\zeta)\lambda$	$-\frac{1}{4}\lambda$	$-\frac{13}{4}\lambda$

TABLE I: Graviton-induced power-law corrections to scalar self-energy and quartic vertex: Summary of the comparison between the conventional approach (with vanishing connection and with general gauge-parameter ζ) and Vilkovisky-DeWitt approach (VDA) (with nonzero connection, and in Landau-DeWitt gauge with $\zeta \rightarrow 0$ in the end). In each entry of the contribution, a common factor $\kappa^2 \mathcal{I}_2$ is factorized out. Our summed results of Γ_2 and Γ_4 on the 3rd column agree with Eqs. (51a)-(51b).

$$\text{Fig.4(b)} = -3\kappa^2 m^2 \mathcal{I}_2, \quad (49b)$$

$$\text{Fig.4(c)} = \left(-1 + \frac{1}{4\zeta}\right) \kappa^2 p^2 \mathcal{I}_2, \quad (49c)$$

$$\text{Fig.4(d)} = \kappa^2 p^2 \mathcal{I}_2, \quad (49d)$$

where the λ -term in (49a) comes from the scalar quartic self-interaction alone, which we have included for the comparison with graviton induced corrections and for the convenience of analysis.

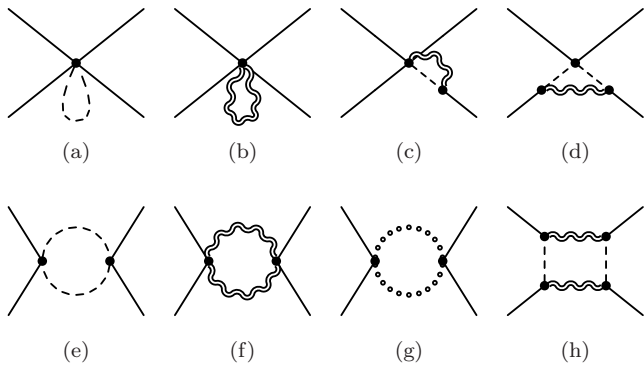


FIG. 5: Graviton-induced corrections to quartic scalar vertex: dashed-line is fluctuating scalar field, double-wavy-line denotes graviton, and circle-dotted-line depicts graviton-ghost.

For the graviton-induced vertex corrections in Fig. 5,

only the first two diagrams have quadratical divergence,

$$\text{Fig.5(a)} = -\frac{1}{4}\lambda\kappa^2\mathcal{I}_2, \quad (50a)$$

$$\text{Fig.5(b)} = -3\lambda\kappa^2\mathcal{I}_2, \quad (50b)$$

and all other diagrams contain logarithmic divergence at most.

Summing up relevant contributions we deduce the two-point proper self-energy and four-point proper vertex for scalar field,

$$\Gamma_2(p) = \left[\frac{1}{2}\lambda + \frac{1}{8}(p^2 - 26m^2)\kappa^2\right] \mathcal{I}_2, \quad (51a)$$

$$\Gamma_4(p) = -i\lambda - \frac{13}{4}\lambda\kappa^2 \mathcal{I}_2, \quad (51b)$$

where we do not include the graviton-induced logarithmic divergent terms (as in [12]) because they are negligible as compared to the dominant power-law corrections. Note that the sum of (49a)-(49d) explicitly proves the exact cancellation of the $\frac{1}{\zeta}$ gauge-parameter poles, which is a consistency check of our Landau-DeWitt gauge calculation.

To fully understand the results (51a)-(51b), we have compared them with those in the conventional BFM (or the equivalent diagrammatical) approach where all connection-induced new terms are set to zero. The findings are summarized in Table-1.

From Eqs. (51a)-(51b), we derive the renormalization for scalar coupling, $\lambda(\mu) = Z_\phi^2 Z_\lambda^{-1} \lambda(\Lambda)$, with the fol-

lowing renormalization constants,

$$Z_\phi = 1 + \frac{1}{128\pi^2} \kappa^2 (\Lambda^2 - \mu^2), \quad (52a)$$

$$Z_\lambda = 1 + \frac{13}{64\pi^2} \kappa^2 (\Lambda^2 - \mu^2), \quad (52b)$$

under the minimal subtraction scheme. Then, with (52) we compute the graviton-induced scalar β -function,

$$\Delta\beta(\lambda, \mu) = +\frac{3\lambda}{8\pi^2} (\kappa^2 \mu^2), \quad (53)$$

which is *not* asymptotically free, contrary to the gauge coupling β -function (31) we derived earlier. The pure scalar loop correction is logarithmic divergent and its renormalization gives the usual non-asymptotically free scalar β -function $\beta_0 = +\frac{3\lambda^2}{16\pi^2}$.

From the two-point proper self-energy (51a), we further perform the renormalization for scalar mass in the on-shell scheme, which fixes the mass counter term,

$$\delta m^2 = \frac{1}{32\pi^2} \left(-\lambda + \frac{25}{4} m^2 \kappa^2 \right) \Lambda^2, \quad (54)$$

where we have defined the renormalized mass, $m_r^2 = m_0^2 - \delta m^2$, and also included the contribution from the pure scalar loop. Comparing the two terms on the right-hand-side of (54), we note that the graviton-induced quadratical divergence is actually much softer since the product $\kappa^2 \Lambda^2 = 16\pi (\Lambda/M_P)^2 = O(10^2)$ for an ultraviolet cutoff $\Lambda \sim M_P$.

It is straightforward to extend the above analysis to the SM Higgs boson, since graviton coupling to scalar fields is universal. Let us write down the SM Higgs doublet,

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi_1 + i\pi_2 \\ \sigma + i\pi_3 \end{pmatrix}, \quad (55)$$

where $\Phi^\dagger \Phi = \sigma^2 + \sum_{a=1}^3 \pi_a^2$ and $\sigma = \bar{\sigma} + \hat{\sigma}$, with $\hat{\sigma}$ being

the SM Higgs boson and $\pi_{1,2,3}$ the would-be Goldstone bosons. The Higgs background field $\bar{\sigma}$ will equal the vacuum expectation value (VEV), v , at the minimum of Higgs potential.

Consider the SM Higgs potential, $V = m^2 (\Phi^\dagger \Phi) + \lambda (\Phi^\dagger \Phi)^2$, with $m^2 < 0$ and the VEV, $v = \sqrt{-m^2/\lambda}$. Then we recompute the gravitational power-law corrections to Higgs boson self-energy and quartic vertex in Fig. 4-5, and deduce the following,

$$\Gamma_{H2}(p) = \left[\frac{1}{2} \lambda + \frac{1}{2} (p^2 - 8m^2) \kappa^2 \right] \mathcal{I}_2, \quad (56a)$$

$$\Gamma_{H4}(p) = -i\lambda - 24\lambda \kappa^2 \mathcal{I}_2. \quad (56b)$$

With these we derive the graviton-induced contributions to the Higgs boson β -function and the Higgs mass counter term,

$$\Delta\beta(\lambda, \mu) = \frac{3\lambda}{8\pi^2} (\kappa^2 \mu^2), \quad (57a)$$

$$\delta m_H^2 = \frac{1}{32\pi^2} (-12\lambda + 7m_H^2 \kappa^2) \Lambda^2, \quad (57b)$$

where m_H is the physical Higgs boson mass. We see that (57a) happens to be the same as in (53) while (57b) has different coefficients from (54). As compared to the single real scalar case, the changes in the computation of (57a)-(57b) arise from two sources: (i) the Feynman rule for the quartic Higgs vertex in the SM has an extra factor 6 relative to that from the real scalar potential below Eq. (43); (ii) there are additional Goldstone loops in the SM which contribute to Fig. 4(a) and Fig. 5(a).

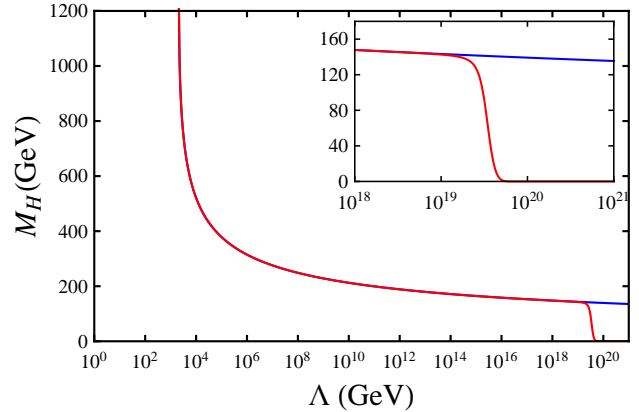


FIG. 6: Gravitational power-law corrections to the triviality bound on the SM Higgs boson mass m_H , where the region above each curve is excluded. The red curve includes the graviton-induced corrections and the blue curve depicts the bound with SM interactions alone.

Note that the conventional Higgs β -function in the SM receives only logarithmic corrections, $\beta_0(\lambda, \mu) = +\frac{3}{2\pi^2} \lambda^2$, and is not asymptotically free. So, with (57a) we can write the summed Higgs β function,

$$\beta(\lambda, \mu) = \tilde{b}_0 \lambda^2 + \tilde{a}_0 \lambda \kappa^2 \mu^2, \quad (58)$$

with $(\tilde{b}_0, \tilde{a}_0) = (\frac{3}{2\pi^2}, \frac{3}{8\pi^2}) > 0$. Solving (58) we deduce the running Higgs coupling $\lambda(\mu)$,

$$\frac{e^{\frac{\tilde{a}_0}{2} \kappa^2 \mu^2}}{\lambda(\mu)} = \frac{e^{\frac{\tilde{a}_0}{2} \kappa^2 \mu_0^2}}{\lambda(\mu_0)} - \tilde{b}_0 \int_{\mu_0}^{\mu} e^{\frac{\tilde{a}_0}{2} \kappa^2 x^2} \frac{dx}{x}. \quad (59)$$

We see that when the right-hand-side (RHS) of (59) vanishes, the renormalized coupling $\lambda(\mu)$ blows up at the Landau pole $\mu = \Lambda_L$,

$$\lambda^{-1}(\mu_0) = \tilde{b}_0 e^{-\frac{\tilde{a}_0}{2} \kappa^2 \mu_0^2} \int_{\mu_0}^{\Lambda_L} e^{\frac{\tilde{a}_0}{2} \kappa^2 x^2} \frac{dx}{x}. \quad (60)$$

This means that the SM as an effective theory must have an UV cutoff $\Lambda < \Lambda_L$. For a given Higgs boson mass $m_H^2 = 2\lambda(m_H)v^2$, let us set $\mu_0 = m_H$. Thus, from (60) we can derive the triviality bound for Higgs boson mass,

$$m_H^2 < \frac{2v^2}{\tilde{b}_0} e^{\frac{\tilde{a}_0}{2} \kappa^2 m_H^2} \left[\int_{m_H}^{\Lambda} e^{\frac{\tilde{a}_0}{2} \kappa^2 x^2} \frac{dx}{x} \right]^{-1}, \quad (61)$$

where $e^{\frac{\tilde{a}_0}{2}\kappa^2 m_H^2} \simeq 1$ holds to high accuracy due to the tiny factor $\frac{\tilde{a}_0}{2}\kappa^2 m_H^2 = \frac{3}{\pi} \frac{m_H^2}{M_P^2} \simeq 0$. It is clear that in the $\tilde{a}_0 \rightarrow 0$ limit, the condition (61) reduces to the familiar triviality bound in the pure SM [28],

$$m_H^2 \ln \frac{\Lambda^2}{m_H^2} < \frac{8\pi^2 v^2}{3}. \quad (62)$$

Keeping this in mind, it is instructive to rewrite our graviton-corrected triviality bound (61) as follows,

$$m_H^2 \ln \frac{\Lambda^2}{m_H^2} < \frac{8\pi^2 v^2}{3(1+X)}, \quad (63a)$$

$$X \equiv \int_{m_H}^{\Lambda} \left[e^{\frac{\tilde{a}_0}{2}\kappa^2 x^2} - 1 \right] \frac{dx}{x} / \ln \frac{\Lambda}{m_H} > 0, \quad (63b)$$

where an overall factor $e^{\frac{\tilde{a}_0}{2}\kappa^2 m_H^2} \simeq 1$ on the RHS of (63a) is safely ignored. We find that, because the integral $X > 0$ generally holds, the gravitational power-law corrections always reduce the RHS of (63a), and thus further *tighten the triviality bound* relative to (62) of the pure SM. The graviton-induced corrections play a dominant role to enhance the triviality bound for the cut-off scale $\Lambda \sim M_P$, as clearly shown in Fig. 6. A systematical expansion of the present section (including the power-law corrections to Yukawa couplings) will be given in Ref. [13].

6. CONCLUSIONS

The fundamental gravitational force universally couples to all the SM particles and can be described by the well-defined perturbation expansion in the modern effective theory formulation [3]. The Vilkovisky-DeWitt method [8] profoundly modifies the conventional BFM, and provides the manifestly gauge-invariant effective action for reliably computing quantum gravity effects. In this work, we used the Vilkovisky-DeWitt method to derive *the first gauge-invariant nonzero gravitational power-law corrections* to the running of gauge couplings. We found the gravitational power-law corrections to be universal, making both Abel and non-Abel gauge couplings asymptotically free [cf. Eq. (31) and analyses at the end of Sec. 3]. We have demonstrated that the graviton-induced power-law runnings always drive the three SM gauge forces toward to the UV fixed point, reaching final unification around the Planck scale and irrespective of the detail of logarithmic corrections (cf. Fig. 3). This raises the conventional GUT scale by three orders of magnitude, and opens up a natural possibility of simultaneous unification of all four fundamental gauge forces at the Planck scale. We further analyzed the power-law corrections to the β -function and mass of the SM Higgs boson [cf. Eqs. (57a)-(57b)]. We found that the

graviton-induced scalar β -function is not asymptotically free, and therefore further tightens the triviality bound on the Higgs boson mass, as shown in Eq. (63) and Fig. 6. Further extensions of the present analysis for computing the power-law corrections will be given elsewhere [13].

Acknowledgments

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