DYNAMICAL ISSUES: SEMIMAJOR AXES

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Outline

- Long term stability of planetary systems
- A few points on the stability of the solar system
- Known exo-planets: semimajor axis distribution
- Multiple exo-planet systems — orbital diversity
- Final stages of planetary orbital evolution — origin of orbital diversity
- Conclusions/Speculations

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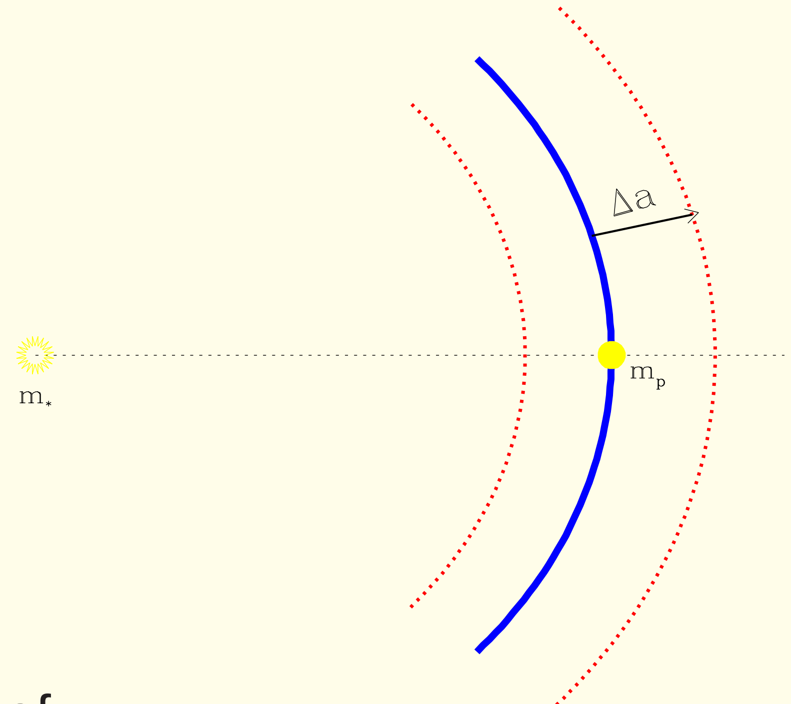
“Planetary architecture is determined by long term stability”

$\Longrightarrow$ (roughly) well separated orbits

- How to quantify “well separated”
  - depends upon m, a, e
  - stability criteria for 1 planet system
    Hill stability: $|\Delta a|/a_{\rm p} > 2.4(m_{\rm p}/m_\star)^{\frac{1}{3}} \simeq 3R_{\rm H}$
    Resonance overlap: $|\Delta a|/a_{\rm p} > 1.5(m_{\rm p}/m_\star)^{\frac{2}{7}}$
    Eccentric planet: $r \ni (q - 3R_{\rm H}, Q + 3R_{\rm H})$
  - secular stability in 2 planet systems
    (see poster)



- No global stability criterion for few-body systems
  - few-body phase space is an intricate mix of chaotic orbits and stable orbits
  - Stability is not monotonic function of separation

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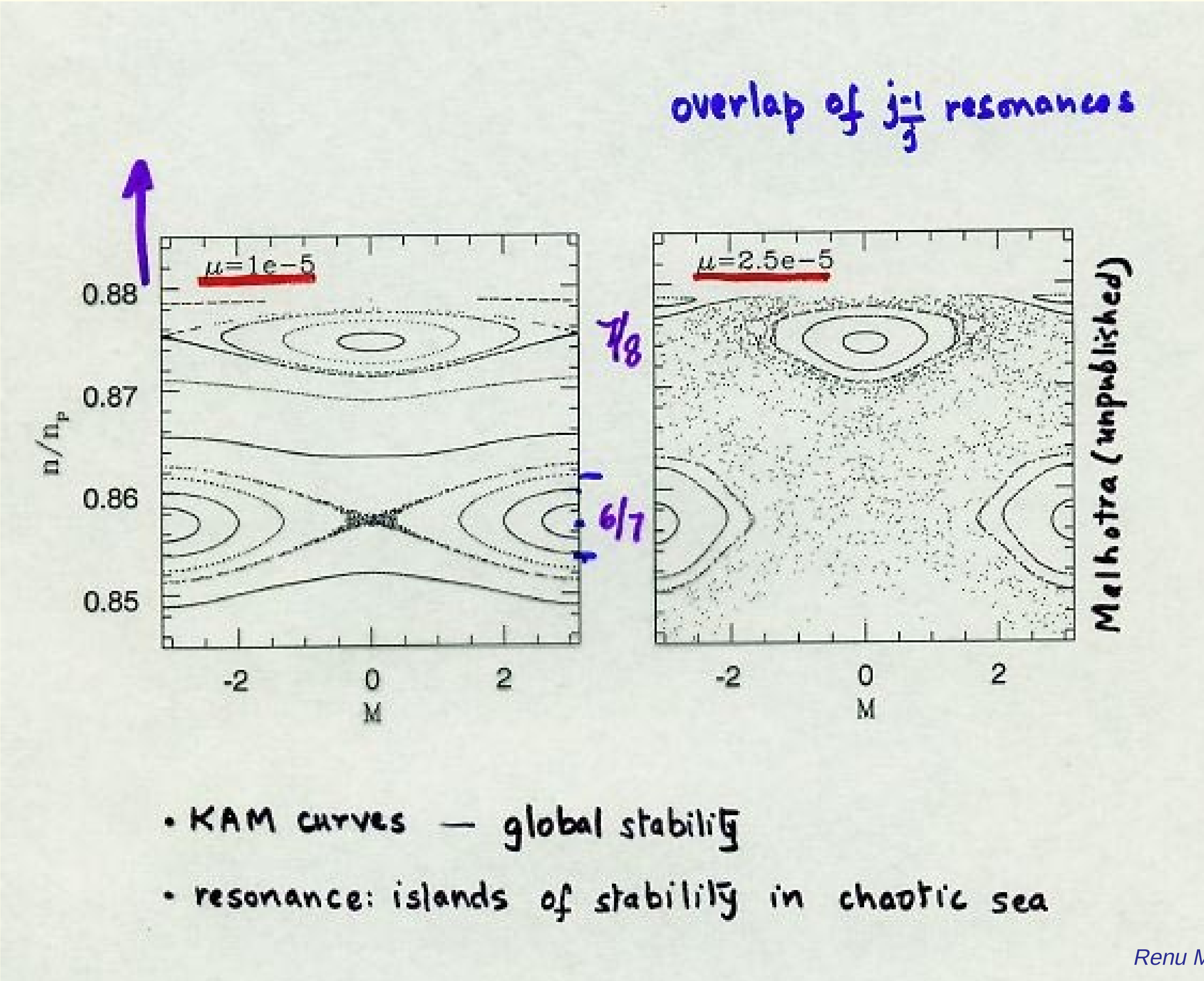
Resonance overlap in the 3 body problem
planar, circular, restricted



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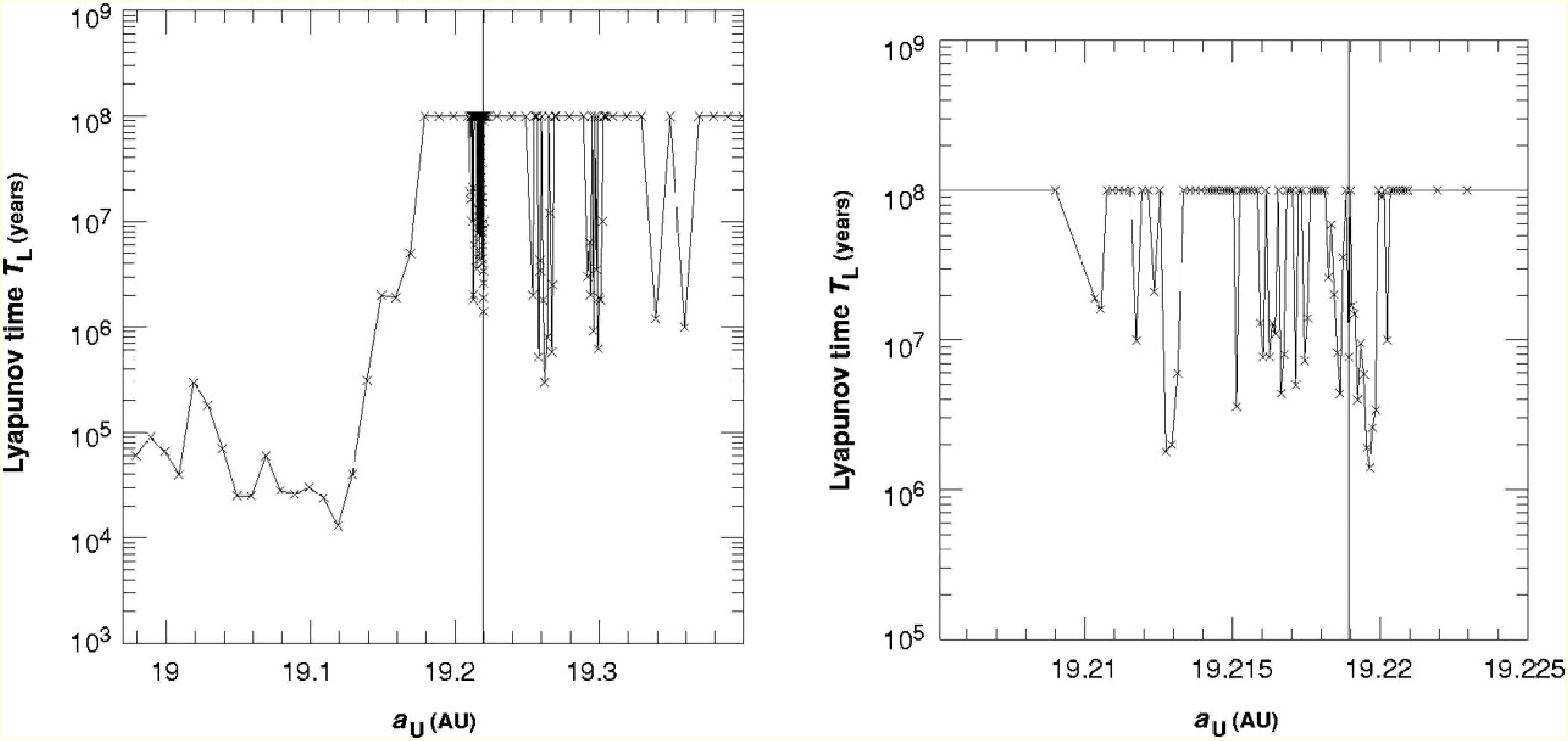
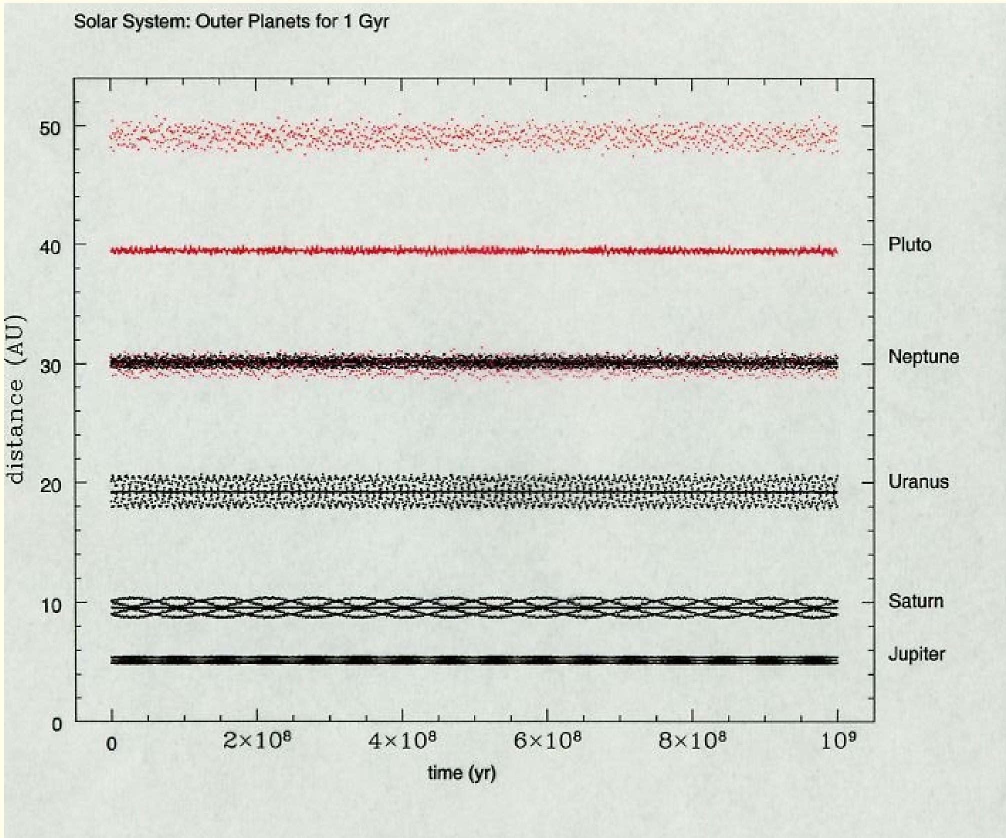
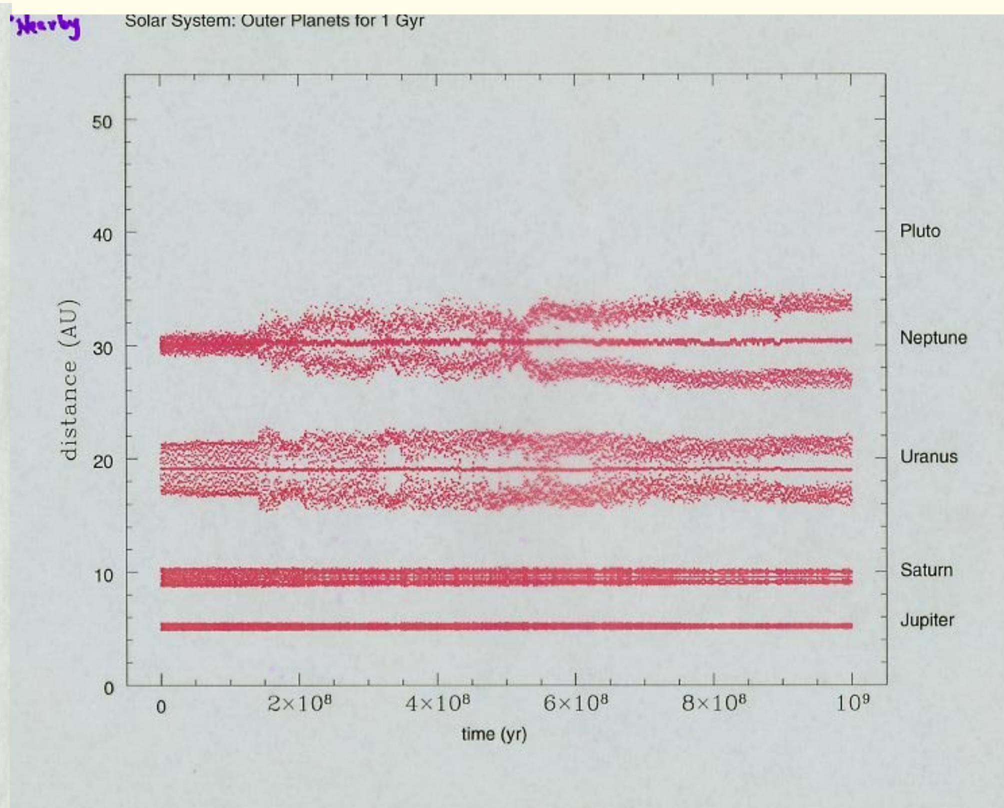
Chaos in Solar system outer planets



from Murray & Holman 1999

The Solar system for 1 Gigayear





A solar system in 'nearby' phase space – long term stable (i.e. no orbit-crossings) but not as 'quiescent' as the actual solar system

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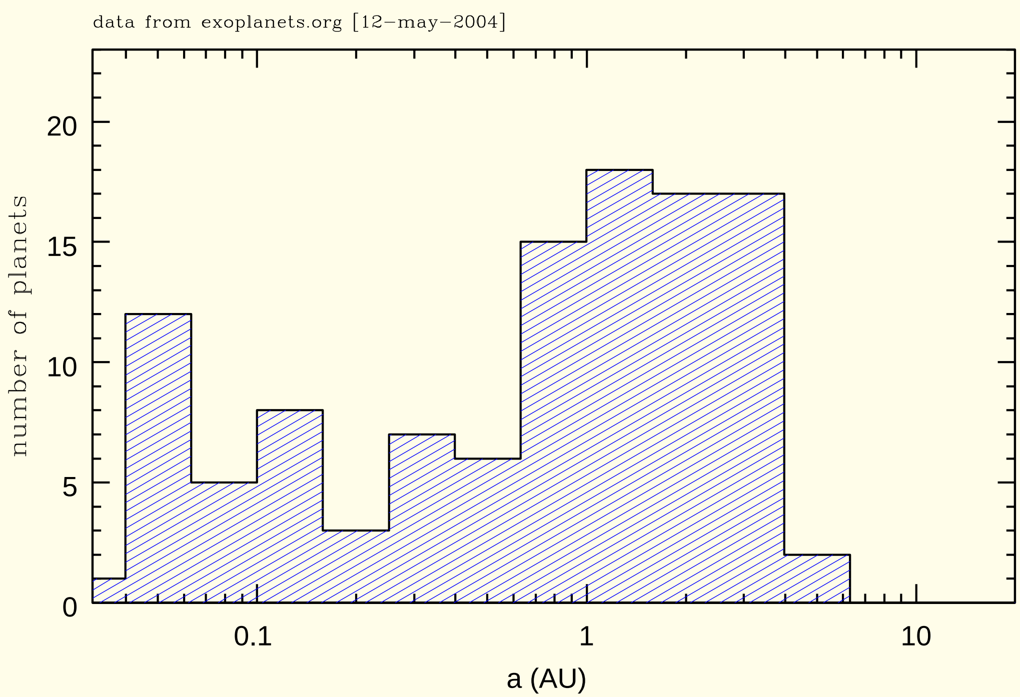
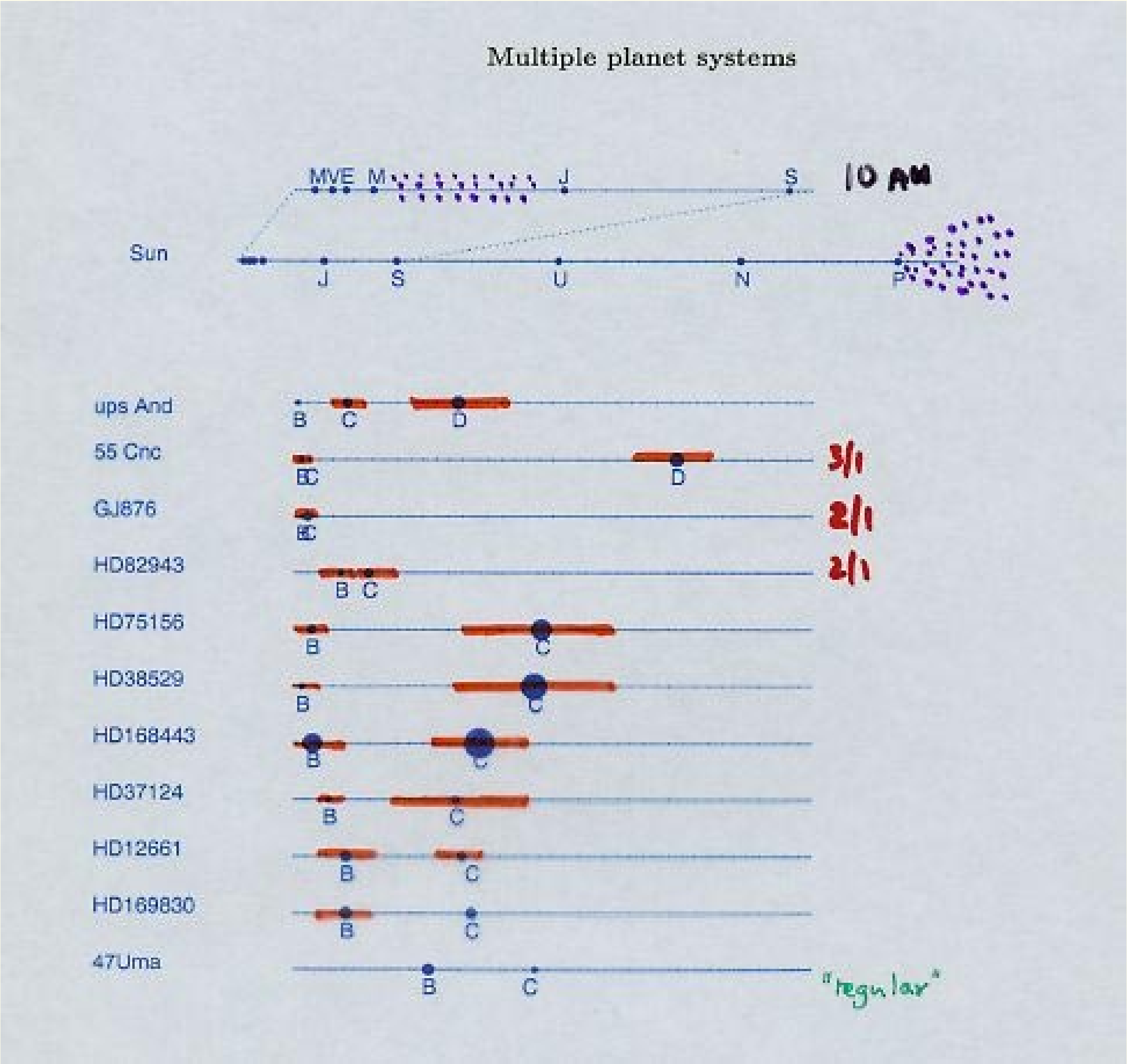
Semimajor axis distribution of known planets



Two classes:

- hot jupiters – *in situ formation or migration?*
- 'normal' giant planets – *late(r) formation, therefore little or no migration?*
  – Implicates diversity of protoplanetary disk (gas) masses.

Multiple planet systems



- diverse architectures!
- resonant planets
  orbital migration in gas disk?
- regular systems
  low eccentricity orbits
- irregular systems
  high eccentricity orbits

Observed diversity suggests that giant planets do not form in well-separated orbits. Orbital evolution owing to planet-planet/debris (chaotic) interactions leads naturally to diverse outcomes.

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Final stages of planetary system formation

gas-free environment
not to forget the small bodies!

Two extrema of evolutionary paths can be identified:

- **planet-planetesimal disk interactions**: migration as in our solar system — planet eccentricities remain damped while semimajor axes spread out

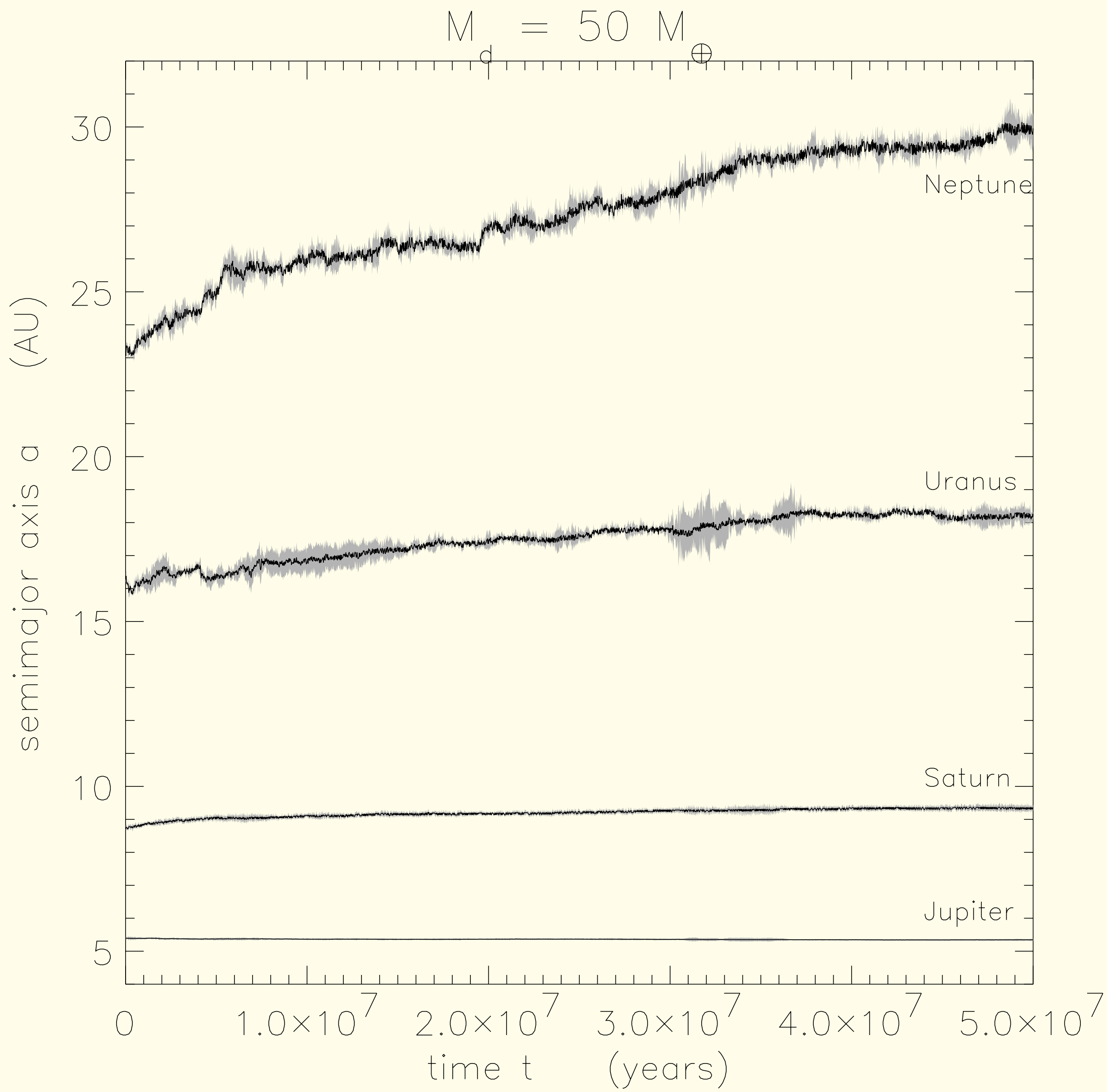
  figure: Hahn & Malhotra 1999 $\Longrightarrow$ p. 9

  Evidence in the remains of solar system debris: Kuiper Belt, Oort Cloud

- **planet-planet interactions**: if the debris disk mass is insufficient, then planet eccentricities will not remain damped, crossing orbits will develop and lead to mergers, scattering, ejection

  figure: Ford et al 2001 $\Longrightarrow$ p. 10

Planet migration via planetesimal debris disk



from Hahn & Malhotra 1999

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Planet-planet scattering

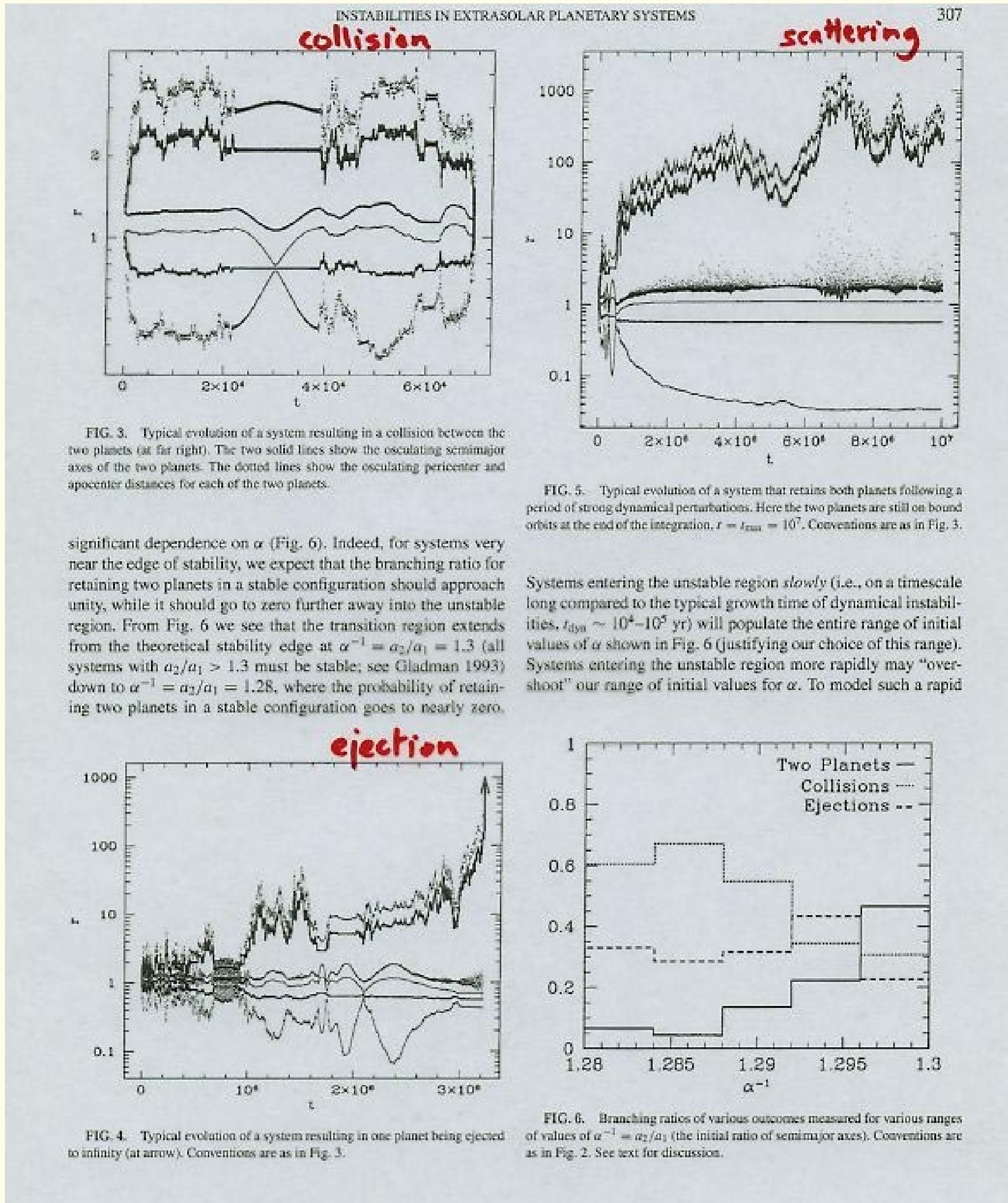


FIG. 3. Typical evolution of a system resulting in a collision between the two planets (at far right). The two solid lines show the osculating semimajor axes of the two planets. The dotted lines show the osculating pericenter and apocenter distances for each of the two planets.

FIG. 5. Typical evolution of a system that retains both planets following a period of strong dynamical perturbations. Here the two planets are still on bound orbits at the end of the integration, $t = t_{max} = 10^7$. Conventions are as in Fig. 3.

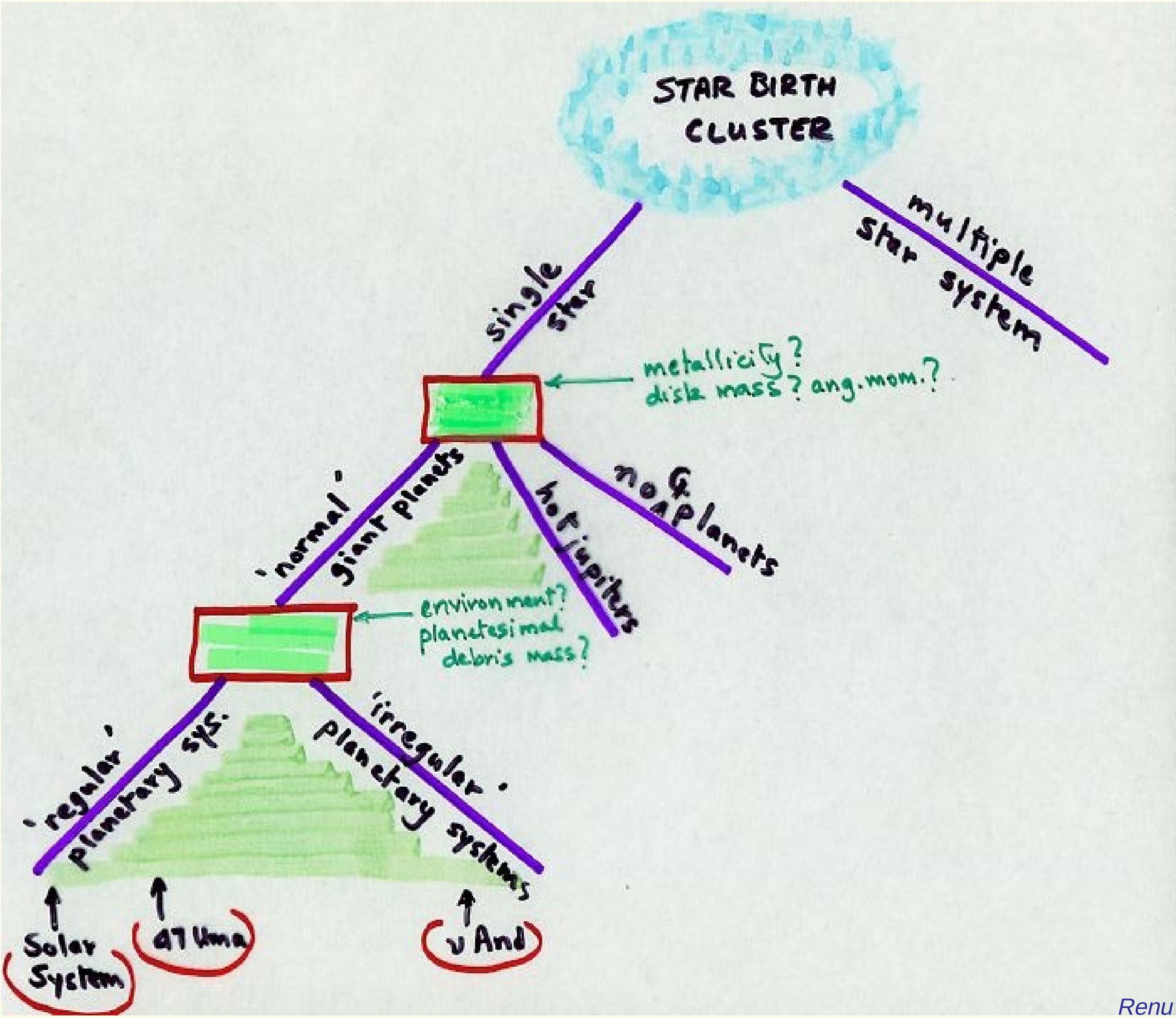
significant dependence on α (Fig. 6). Indeed, for systems very near the edge of stability, we expect that the branching ratio for retaining two planets in a stable configuration should approach unity, while it should go to zero further away into the unstable region. From Fig. 6 we see that the transition region extends from the theoretical stability edge at $\alpha^{-1} = a_2/a_1 = 1.3$ (all systems with $a_2/a_1 > 1.3$ must be stable; see Gladman 1993) down to $\alpha^{-1} = a_2/a_1 = 1.28$, where the probability of retaining two planets in a stable configuration goes to nearly zero. Systems entering the unstable region *slowly* (i.e., on a timescale long compared to the typical growth time of dynamical instabilities, $t_{dyn} \sim 10^4$–10^5 yr) will populate the entire range of initial values of α shown in Fig. 6 (justifying our choice of this range). Systems entering the unstable region more rapidly may "overshoot" our range of initial values for α. To model such a rapid

FIG. 4. Typical evolution of a system resulting in one planet being ejected to infinity (at arrow). Conventions are as in Fig. 3.

FIG. 6. Branching ratios of various outcomes measured for various ranges of values of $\alpha^{-1} = a_2/a_1$ (the initial ratio of semimajor axes). Conventions are as in Fig. 2. See text for discussion.

from Ford et al 2001

Summary: A flowchart of planetary system formation



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