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Secular increase of the Astronomical Unit: a possible explanation in terms of the total angular momentum conservation law

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ABSTRACT

Aims. We show a possible explanation for the recently reported secular increase of the Astronomical Unit (AU) by Krasinsky and Brumberg (2004).

Methods. The mechanism proposed is analogous to the tidal acceleration in the Earth-Moon system, which is based on the conservation of the total angular momentum and we apply this scenario to the Sun-planets system.

Results. Assuming the existence of some tidal interactions that transfer the rotational angular momentum of the Sun and using reported value of the positive secular trend in the astronomical unit, $\frac{d}{dt}\mathrm{AU} = 15 \pm 4$ (m/cy), the suggested change in the period of rotation of the Sun is about 3 ms/cy in the case that the orbits of the eight planets have the same "expansion rate." This value is sufficiently small, and at present it seems there are no observational data which exclude this possibility. Effects of the change in the Sun's moment of inertia is also investigated. It is pointed out that the change in the moment of inertia due to the radiative mass loss by the Sun may be responsible for the secular increase of AU, if the orbital "expansion" is happening only in the inner planets system.

Key words. celestial mechanics – ephemerides – astronomical unit

1. Introduction

The Astronomical Unit (hereafter we abbreviate AU) is one of the most essential scale in astronomy which characterizes the scale of the solar system and the standard of cosmological distance ladder. AU is also the fundamental astronomical constant that associates two length unit; one (m) in International System (SI) of Units and one (AU) in Astronomical System of Units.

In the field of fundamental astronomy e.g., the planetary ephemerides, it is one of the most important subjects to evaluate AU from the observational data. The appearance of planetary radar and spacecraft ranging techniques has led to great improvement of determination of AU. Modern observations of the major planets, including the planetary exploration spacecrafts such as Martian landers and orbiters, make it possible to control the value of AU within the accuracy of one meter or even better. Actually present best-fit value of AU is obtained as (Pitjeva 2005),

$$1\,(\mathrm{AU}) = \mathrm{AU}\,(\mathrm{m}) = 1.495978706960 \times 10^{11} \pm 0.1\,(\mathrm{m}). \quad (1)$$

However, recently Krasinsky and Brumberg reported the positive secular trend in AU as $\frac{d}{dt}\mathrm{AU} = 15 \pm 4$ (m/cy) [1] from the analysis of radar ranging of inner planets and Martian landers and orbiters, see Krasinsky and Brumberg (2004) and also Standish (2005). This value is about 100 times larger than the present determination error of AU, see Eq. (1). This secular increase of AU was discovered by using following formula (Krasinsky 2007),

$$t_{\mathrm{theo}} = \frac{d_{\mathrm{theo}}}{c}\left[\mathrm{AU} + \frac{d\mathrm{AU}}{dt}(t - t_0)\right], \quad (2)$$

in which t_{theo} is the theoretical value of round-trip time of radar signal in the SI second, d_{theo} is interplanetary distance obtained from ephemerides in the unit of (AU), c is the speed of light in vacuum, AU and $d\mathrm{AU}/dt$ are, respectively, the astronomical unit as Eq. (1) and its time variation, and t_0 is the initial epoch of ephemerides. t_{theo} is compared with the observed lapse time of signal t_{obs} and then AU, $d\mathrm{AU}/dt$ are fit by the least square method.

The time dependent term $(d\mathrm{AU}/dt)(t - t_0)$ in Eq. (2) currently cannot be related with any theoretical predictions, hence several attempts have been made to explain this secular increase of AU, including e.g., the effects of the cosmic expansion (Krasinsky and Brumberg 2004, Mashhoon et al. 2007, Arakida 2009), mass loss of the Sun (Krasinsky and Brumberg 2004, Noerdlinger 2008), the time variation of gravitational constant G (Krasinsky and Brumberg 2004), the influence of dark matter (Arakida 2008) and so on. But unfortunately so far none of them seems to be successful.

In this paper, we will take an another viewpoint and give an explanation for the secular increase of AU, based on the standard conservation law of the total angular momentum. The mechanism is closely analogous to the case of the tidal acceleration in the Earth-Moon system which induces the transfer of the rotational angular momentum of the Earth into the orbital angular momentum of the Moon due to tidal friction. We will apply similar scenario to the Sun-planets system.

[1] In this paper, cy means the century according to Krasinsky and Brumberg (2004).

2. Tidal acceleration in the Earth-Moon system: a brief summary

In this section, we briefly summarize the tidal acceleration in the Earth-Moon system. See e.g., Chapter 4 of Murray and Dermott (1999) for details.

The conservation law of the total angular momentum in the Earth-Moon system is

$$\frac{d}{dt}(\ell_{\rm E} + L_{\rm M}) = 0, \tag{3}$$

where $\ell_{\rm E}$ is the rotational angular momentum of the Earth, $L_{\rm M}$ is the orbital angular momentum of the Moon, and for simplicity we have neglected the rotational angular momentum of the Moon, which is about 10^{-5} times smaller than $L_{\rm M}$. The rotational angular momentum is written in terms of the moment of inertia $I_{\rm E}$ and the period of rotation $T_{\rm E}$ as

$$\ell_{\rm E} = I_{\rm E}\frac{2\pi}{T_{\rm E}}. \tag{4}$$

The moment of inertia can be written as

$$I_{\rm E} = \frac{2}{5}\gamma M R^2, \tag{5}$$

where M is the mass and R is the radius of the Earth, and γ is a constant parameter of $O(1)$ which may depend on the density inhomogeneity inside the Earth. In the case of the perfect sphere with uniform density inside, γ is unity. In cases where the inner density is higher than the outer one, γ is in general less than unity. Often quoted value for the Earth is $\gamma \simeq 0.83$. Under the assumption that the Moon's orbit is circular with radius r, the orbital angular momentum is written as

$$L_{\rm M} = mrv = m\sqrt{GMr}, \tag{6}$$

where m is the mass of the Moon, G is the gravitational constant.

Then, if we assume $I_{\rm E}$, m, and M are constant, the conservation of the total angular momentum Eq. (3) reads

$$\frac{\dot{T}_{\rm E}}{T_{\rm E}} = \frac{1}{2}\frac{L_{\rm M}}{\ell_{\rm E}}\frac{\dot{r}}{r} \tag{7}$$

The motion of the Moon can be traced with an accuracy of a few centimeters by the lunar laser ranging experiments. The measurements yield the numerical value $\dot{r} = +3.82 \pm 0.07$ (m/cy) (Dickey et al. 1994) then

$$\frac{\dot{r}}{r} \simeq 1.0 \times 10^{-8}\,({\rm cy}^{-1}). \tag{8}$$

Using Eq. (8), the numerical value of Eq. (7) is

$$\frac{\dot{T}_{\rm E}}{T_{\rm E}} \simeq 2.0 \times \gamma^{-1} \times 10^{-8}\,({\rm cy}^{-1}), \tag{9}$$

or equivalently, $\dot{T}_{\rm E} \simeq 2.1$ (ms/cy), where we have used $\gamma = 0.83$.

On the other hand, historical records over 2700 years can be used to compute the change in the length of the day, and the following average value is found:

$$\dot{T}_{\rm obs} = +1.70 \pm 0.05\,({\rm ms/cy}). \tag{10}$$

The gradual slowing of the Earth's rotation is due to the tidal force between the orbiting Moon and the Earth, or the tidal friction. It is explained that the discrepancy between the estimated value $\dot{T}_{\rm E}$ and the observed one $\dot{T}_{\rm obs}$ is largely due to the change in the moment of inertia of the Earth caused by the melting of ice at the poles.

3. Application to the Sun-planets system

In this section, we apply the same argument in the previous section to the Sun-planets system. All we need is the conservation of the total angular momentum in the solar system. For the sake of simplicity, we assume that each planet, denoted by subscript i, has a circular orbit with radius r_i. The length AU, denoted by a, is used to normalize r_i, then for the Earth's radius $r_{\rm E} = a$, and for the moment it is assumed that the orbits of the all planets have the same "expansion rate," i.e.,

$$\frac{\dot{r}_i}{r_i} = \frac{\dot{a}}{a} \tag{11}$$

for each i.

In the case that the Sun's moment of inertia does not change, from the analogy to Eq. (7), we obtain the change in the period of rotation of the Sun $T_\odot$ as

$$\frac{\dot{T}_\odot}{T_\odot} = \frac{1}{2}\frac{L}{\ell_\odot}\frac{\dot{a}}{a}, \tag{12}$$

where L is the sum of the orbital angular momentums of all planets

$$L = \sum_i m_i\sqrt{GM_\odot r_i}, \tag{13}$$

$\ell_\odot$ is the rotational angular momentum of the Sun

$$\ell_\odot = I_\odot\frac{2\pi}{T_\odot} = \frac{2}{5}\gamma_\odot M_\odot R_\odot^2\frac{2\pi}{T_\odot}, \tag{14}$$

and $M_\odot$ and $R_\odot$ are the mass and the radius of the Sun, respectively, and $\gamma_\odot$ is again a constant parameter of $O(1)$.

Using the value $\dot{a} \simeq 15$(m/cy) reported by Krasinsky and Brumberg, $\dot{a}/a \simeq 1.0 \times 10^{-10}$ and the right-hand-side of Eq. (12) is evaluated as

$$\frac{\dot{T}_\odot}{T_\odot} \simeq 1.4 \times \gamma_\odot^{-1} \times 10^{-9}\,({\rm cy}^{-1}). \tag{15}$$

If we use the value as $\gamma_\odot = 1$ and the rotational period of the Sun as $T_\odot = 25.38$ (days), the estimated value of the change in $T_\odot$ is

$$\dot{T}_\odot \simeq 3\,({\rm ms/cy}). \tag{16}$$

The estimated value is sufficiently small and seems well within the observational limit. Even if the rotational period continues to change at the same rate for, say, 5 billion years (the age of the Sun), the amount of change in the period $\Delta T_\odot/T_\odot$ is just about 7%.

4. Effects of the change in the moment of inertia

In the previous section, we have considered only the case that the moment of inertia $I_\odot$ is constant, namely,

$$\frac{d}{dt}\ell_\odot = \frac{d}{dt}\left(I_\odot\frac{2\pi}{T_\odot}\right) = -\frac{\dot{T}_\odot}{T_\odot}\ell_\odot\,. \tag{17}$$

If we generalize our result to the case that $I_\odot$ and $M_\odot$ are not constant, Eq. (12) changes to

$$-\frac{\dot{\gamma}_\odot}{\gamma_\odot} - \frac{\dot{M}_\odot}{M_\odot} - 2\frac{\dot{R}_\odot}{R_\odot} + \frac{\dot{T}_\odot}{T_\odot} = \frac{1}{2}\frac{L}{\ell_\odot}\left(\frac{\dot{M}_\odot}{M_\odot} + \frac{\dot{a}}{a}\right). \tag{18}$$

As a first approximation, we assume that the radiative mass loss occurs isotropically along radial direction and does not carry the angular momentum.

The first term in the left-hand-side of Eq. (18) represents the effect of change in the internal density distribution of the Sun, and so far we do not have enough information on it in detail.

The second term in the left-hand-side of Eq. (18) represents the effect of mass loss, which can be evaluated in the following way. The Sun has luminosity at least 3.939×10^{26} W, or 4.382×10^{9} kg/s, including electromagnetic radiation and contribution from neutrinos (Noerdlinger 2008). The particle mass loss rate by the solar wind is about 1.374×10^{9} kg/s, according to Noerdlinger (2008). The total solar mass loss rate is then

$$-\frac{\dot{M}_{\odot}}{M_{\odot}} = 9.1 \times 10^{-12}\,(\mathrm{cy}^{-1})\,, \tag{19}$$

which is less than a hundredth of the required value to explain the secular increase of AU (see Eq. (15)). Therefore, we can conclude that the solar mass loss term in the left-hand-side of Eq. (18) does not make a significant contribution to the secular increase of AU, if the orbits of the eight planets have the same "expansion rate."

Note that the term which is proportional to $\dot{M}_{\odot}/M_{\odot}$ also appears in the right-hand-side of Eq. (18). This term may be called as the Noerdlinger effect (Noerdlinger 2008). Noerdlinger (2008) already investigated this effect of solar mass loss, and concluded that the effect can only account for less than a tenth of the reported value by Krasinsky and Brumberg.

The third term in the left-hand-side of Eq. (18) is the contribution from the change in the solar radius. Although the very short-time and small-scale variability in the solar radius may be actually observed in the context of helioseismology, we have no detailed information on the secular change in $R_{\odot}$ so far.

5. Case of the Sun-inner planets system

In the previous sections, we have assumed that the orbits of the all planets have the same "expansion rate", i.e., Eq. (11). However, the recent positional observations of the planets with high accuracy are mostly done within the inner planets region. Those precise measurements for the position of the inner planets have revealed the positive secular trend in AU. Therefore, it is instructive to consider the case that the "expansion rates" of the planetary orbits are not homogeneous but inhomogeneous in the sense that only the orbits of the inner planets expand.

In this section, we consider the case that the "expansion" of the planetary orbit occurs only for the inner planets.

In this case, the sum of the orbital angular momentum of all planets L is replaced by the sum of the inner planets L_{in}:

$$L_{\mathrm{in}} \equiv \sum_{i=1}^{4} m_i \sqrt{G M_{\odot} r_i}. \tag{20}$$

Then Eq. (18) is now

$$-\frac{\dot{\gamma}_{\odot}}{\gamma_{\odot}} - \frac{\dot{M}_{\odot}}{M_{\odot}} - 2\frac{\dot{R}_{\odot}}{R_{\odot}} + \frac{\dot{T}_{\odot}}{T_{\odot}} = \frac{1}{2}\frac{L_{\mathrm{in}}}{\ell_{\odot}}\left(\frac{\dot{M}_{\odot}}{M_{\odot}} + \frac{\dot{a}}{a}\right). \tag{21}$$

Note that the sum of the angular momentum of inner planets amounts only 0.16% of the total L:

$$\frac{L_{\mathrm{in}}}{L} \simeq 1.6 \times 10^{-3}. \tag{22}$$

Therefore, under the assumption that the change in the rotational angular momentum of the Sun affects only the orbital angular momentums of the inner planets, the required values which were calculated in the previous sections to explain the secular increase of AU can now be revised to be 1.6×10^{-3} times smaller. In particular, the right hand side of Eq. (21) is

$$\frac{1}{2}\frac{L_{\mathrm{in}}}{\ell_{\odot}}\frac{\dot{a}}{a} \simeq 2.2 \times 10^{-12}\,(\mathrm{cy}^{-1}). \tag{23}$$

Interestingly, it is the same order of magnitude as (actually it is about 4 times smaller than) $\dot{M}_{\odot}/M_{\odot}$ (see Eq. (19)).

Then we can conclude from Eq. (21) that the decrease of rotational angular momentum of the Sun due to the radiative mass loss already has more than enough contribution to the secular increase of the orbital radius of the inner planets.

6. Conclusion

In this paper, we considered the secular increase of astronomical unit recently reported by Krasinsky and Brumberg (2004), and suggested a possible explanation for this secular trend by means of the conservation law of total angular momentum. Assuming the existence of some tidal interactions that transfer the angular momentum from the Sun to the planets system, we have obtained the following results.

From the reported value $\frac{d}{dt}\mathrm{AU} = 15 \pm 4\,(\mathrm{m/cy})$, we have obtained the required value for the variation of rotational period of the Sun is about 3 (ms/cy), if we assume that eight planets in the solar system experience the same orbital expansion rate. This value is sufficiently small, and at present it seems there are no observational data which exclude this possibility.

Moreover, we have found that the effects of change in the moment of inertia of the Sun due to the radiative mass loss may be responsible for explaining the secular increase of AU. Especially, when we suppose that the orbital expansion occurs only in the inner planets region, the decrease of rotational angular momentum of the Sun has enough contribution to the secular increase of the orbital radius. Then as an answer to the question "why is AU increasing ?", we propose one possibility, namely "because the Sun is losing its angular momentum."

In the process of planetary ephemerides construction, the effects due to both solar mass loss and rotational angular momentum transfer is currently not included. However, as mentioned by Noerdlinger (2008), the solar mass loss induces the variation in the orbital radius of planet $\delta\dot{r} \sim +1$ (m/cy). Furthermore as we showed in this paper, the change of Sun's moment of inertia may also cause the significant contribution to orbital motion of planets. Since the observational accuracy in the solar system is rapidly growing then these two effects need to be included to create the ephemerides and to evaluate the various astronomical constants.

Showed also in previous section, the obtained value in Eq. (23) is practically about 4 times smaller than that due to $\dot{M}_{\odot}/M_{\odot}$. Nonetheless the estimated $d\mathrm{AU}/dt$ in the different data sets and fitting parameters is distributed within the range 7.9 ± 0.2 to 61.0 ± 6.0 (m/cy) then seems not to be tightly constrained, see Table 2 of Krasinsky and Brumberg (2004). Hence we can say that the estimated value in Eq. (23) falls into the suitable result.

In order to investigate the secular increase of AU and ascertain its origin, it is important not only to perform the theoretical researches but also to re-analyze the date and to obtain more accurate value of $d\mathrm{AU}/dt$. adding new observations e.g., Mars Reconnaissance Orbiter, Phoenix and forthcoming MESSENGER which is cruising to the Mercury. Further it seems to be meaningful for tackling this AU issue to use the observations of outer planets as well, such as Cassini, Pioneer 10/11, Voyager 1/2, and New Horizons for Pluto because it is more natural situation that the variation of moment of inertia of the Sun acts whole of planets in solar system. These subjects may be important future works to be investigated.

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