

# Constraining Inverse Curvature Gravity with Supernovae

J. Weller<sup>1</sup>, O. Mena<sup>2</sup> and J. Santiago<sup>2</sup>

<sup>1</sup>*Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, UK.*

<sup>2</sup>*Fermi National Accelerator Laboratory, Batavia, IL 60510-0500, USA.*

We show how modifications of gravity, which involve inverse curvature terms, can fit the observed magnitude - redshift relation of distant type Ia Supernovae. In order to achieve this we have to solve the modified Friedmann equations and we discuss different regimes of the solution, dependent on the free parameters, which lead to accelerated expansion.

## 1 Introduction

Evidence for an accelerated expansion of the Universe has been mounting in recent years <sup>1,2,3,5,6,7,8,4</sup>. While the precision of the data is continually improving, explanations for the cause of accelerated expansion are still in its infancy and at best ad hoc. In principle there are four possible ways to explain the observations: the strong energy condition is violated, i.e. the late Universe; is dominated by a fluid with  $\rho + 3p \leq 0$ , with  $\rho$  the energy density in the fluid and  $p$  its pressure; gravity is modified on large scales and this modification leads to accelerated expansion; the Universe as a whole is not homogeneous and we are happening to be in a bubble, which is expanding in an accelerated fashion; or the data might be wrong. In this talk we concentrate on the second possibility. In a similar fashion as models of Quintessence <sup>9,10,11,12</sup> are motivated by inflationary models, the modification of gravity we discuss here, has been first discussed by Starobinsky for early de Sitter Universes <sup>13</sup>. If we look at the action for gravity

$$S_{E-H} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R + \mathcal{L}_m) ,$$

where  $g$  is the determinant of the metric,  $R$  the Ricci curvature scalar and  $\mathcal{L}_m$  the matter Lagrangian. The curvature term can be generalized to  $R + mR^n$ . Starobinsky noticed that for  $n > 0$  this can lead to de Sitter solutions in the early Universe. In order to obtain acceleration in the late Universe at large scales, where the Universe is approximately flat the only requirement is  $n < 0$ . This has been investigated in <sup>14</sup>. The surprising finding was that an additional  $1/R$  term in the Einstein-Hilbert action would allow for accelerated expansion solutions in the late Universe, which are attractor solutions. In other words they do not require extremely fine tuning of the initial conditions in order to explain the observations. However, it was soon noticed that these models are in serious trouble with solar system tests <sup>15</sup>.

## 2 General Inverse Curvature Models

However, the form of the gravitational action suggested by Starobinsky <sup>13</sup> is not completely general. In general one can try to add any quadratic combination of the curvature scalar, Ricci and Riemann tensor to the action. In order to obtain accelerated expansion the general allowed form of the action is

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( R + \frac{\mu^{4+n}}{(aR^2 + bP + cQ)^n} + \mathcal{L}_m \right) , \quad (1)$$

with  $P \equiv R_{\mu\nu}R^{\mu\nu}$  the square of the Ricci tensor,  $Q \equiv R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$  the square of Riemann tensor and  $G$  Newton's constant. Models with this action have an unstable de Sitter solution and later

time power law acceleration<sup>16</sup>. However, if one introduces explicit dependence on the Riemann tensor into the action the equations of motion become fourth order and there might be ghost-like excitation due to the presence of a massive spin-2 gravitons with negative energy eigenstates. However it was shown in<sup>17</sup> that in case  $P$  and  $Q$  appear in the combination  $4P - Q$ , which interestingly is also realized in Gauss-Bonnet gravity, the massive spin-2 excitations vanish. Furthermore it was shown that for the models, which lead to accelerated expansion today all solar system tests are passed<sup>18</sup>. This is due to the fact that in order to obtain accelerated expansion  $\mu$  has to be chosen roughly of the order of the observed Hubble constant  $H_0$ , ie.  $\mu \approx 10^{-33}$  eV. After the presentation of this talk a paper was submitted<sup>19</sup>, which showed that besides the problem with ghost-like excitations, these models are also threatened by tachionic propagation modes. However there are parameter combination of  $a$ ,  $b$  and  $c$ , where non such modes are present!

### 3 Solving the Modified Friedmann Equation

In order to obtain a theoretical prediction we have to solve the modified Friedman equation for the  $n = 1$  model<sup>20</sup>. For a compact notation we define the following quantities:  $\alpha \equiv \frac{12a+4b+4c}{12a+3b+2c}$ ,  $\hat{\mu} \equiv \frac{\mu}{|12a+3b+2c|^{1/6}}$ ,  $\sigma \equiv \text{sign}(12a + 3b + 2c)$ . We will solve the dynamical system in the variables  $u \equiv \ln(H/\hat{\mu})$ , with  $H = \dot{a}/a$  the Hubble parameter. As time variable we choose e-foldings  $N \equiv \ln a$ . We then obtain for the modified Friedman equation

$$u''\mathcal{P}_1(u') + \mathcal{P}_2(u') + 18\sigma(\mathcal{P}_3(u'))^3 e^{6u}(e^{2(\bar{u}-u)} - 1) = 0, \quad (2)$$

where a prime denotes the derivative with respect to  $N$  and we have defined the following polynomials,  $\mathcal{P}_1(y) = 6\alpha^2 y^2 + 24\alpha y + 32 - 8\alpha$ ,  $\mathcal{P}_2(y) = 15\alpha^2 y^4 + 2\alpha(50 - 3\alpha)y^3 + 4(40 + 11\alpha)y^2 + 24(8 - \alpha)y + 32$ ,  $\mathcal{P}_3(y) = \alpha y^2 + 4y + 4$ . The source is  $\bar{u} \equiv \ln[\bar{\omega}_r \exp(-4N) + \bar{\omega}_m \exp(-3N)]/2$ , where we have defined the appropriately normalized values of the energy densities *today* as  $\frac{8\pi G}{3} \frac{\rho_{r,m,0}}{\hat{\mu}^2} \equiv \bar{\omega}_{r,m}$ , with  $\rho_{r,m}^0$  the present densities in matter and radiation and we have exploited the fact that the energy-momentum tensor is still covariantly conserved. This means that the source in Eqn. (2) corresponds to a matter component with no dark energy. In theory one would solve the Friedmann equation in (2) for arbitrary initial conditions. However since the 2nd order differential equation is stiff and possibly ill-conditioned, this seems impossible to achieve. We therefore took the following approach: We assumed that in order to obtain cosmologies, which are not ruled out by observations, the Universe has to have phases, where it is radiation and matter dominated. Starting from these initial conditions allows one to construct *stable* approximate solution, which exhibit small corrections to the scaling of  $H$  compared to a matter or radiation dominated Universe. However, at late times there are significant deviations from a matter dominated Universe. We found that our approximate solution is valid to within 0.1% for  $z > 7$ . We hence employ the approximate solution for large redshift and then use this solution as an initial condition for the exact numerical solution starting at  $z = 7$ . This is numerically feasible and stable<sup>20</sup>. However we also have to make sure that this method of solving the equations is valid for all possible parameter choices of  $\bar{\omega}_m$ ,  $\alpha$  and  $\sigma$ . In order to classify the different regions, we define the following special values of  $\alpha$ :  $\alpha_1 \equiv 8/9$ ,  $\alpha_2 \equiv 4(11 - \sqrt{13})/27 \approx 1.095$  and  $\alpha_3 \equiv 20(2 - \sqrt{3})/3 \approx 1.786$ . For  $\alpha < \alpha_1$  both signs of  $\sigma$  result in an acceptable (non-singular) dynamical evolution, but nevertheless in a bad fit for Supernovae data. For  $\alpha_1 < \alpha < \alpha_2$  only  $\sigma = -1$  leads to an acceptable expansion history, since for  $\sigma = +1$  a singular point is violently approached in the past. For  $\alpha_2 < \alpha$  the singular point is approached for  $\sigma = -1$ , hence  $\sigma = +1$  is the only physically valid solution. In this latter case, when  $\alpha_2 < \alpha < \alpha_3$ , the system goes to a stable attractor that is decelerated, thus giving a bad fit to SNe data, for  $\alpha < 32/21$  and gets accelerated for larger  $\alpha$ . For  $\alpha_3 < \alpha$  there is no longer a stable attractor and the system smoothly goes to a singularity in the future. That singularity occurs earlier as  $\alpha$  increases so that there is

a limiting function  $\alpha_4(\bar{\omega}_m)$ , at which the singularity is reached today. It is important to stress that this singularity is approached in a very smooth fashion, allowing for a phenomenologically viable behaviour of the system, as opposed to the evolution when the *wrong* value of  $\sigma$  is chosen, where the singularity is hit almost instantaneously. Finally, for values of  $\alpha > 24.9$ , there are stable attractors again but these are never accelerated and the resulting fit to SNe data is not acceptable. To summarize, there are two regions that give a dynamical evolution of the system compatible with SNe data, the *low* region with  $\alpha_1 < \alpha < \alpha_2$ , for which  $\sigma = -1$ , and the *high* region where  $\alpha_2 < \alpha < \alpha_4$ , for which  $\sigma = +1$ .

#### 4 Comparison to Supernovae Data

In order to compare to the observed magnitude-redshift relation we have to work out the luminosity distance in a flat Universe  $d_L(z) = c(1+z) \int_0^z \frac{dz'}{H(z')}$ . In order to fit the theory to the observed magnitude - redshift relation there is an additional ambiguity of choice of the intrinsic magnitude  $\mathcal{M}$  of the Supernovae. This leads in standard gravity to the inability to obtain constraints on  $H_0$  just from SNe. For the modified gravity model this results in the inability to constrain  $\hat{\mu}$ . Taking into account the results from the dynamical analysis we can hence simultaneously fit for  $\alpha$  and  $\bar{\omega}_m$  with the Supernovae data. For the analysis presented here we choose a recent compilation of Supernovae samples by<sup>3</sup>. In Fig. 1 we show the results

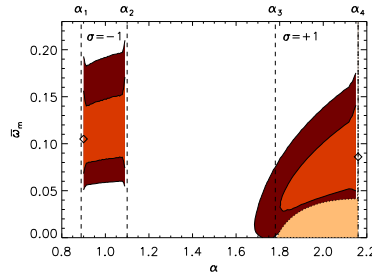


Figure 1: 1 and 2- $\sigma$  joint likelihoods on  $\bar{\omega}_m$  and  $\alpha$ . In the *low* region  $\sigma = -1$  whereas in the *high* region  $\sigma = +1$ . The shaded area on the right determines the region  $\alpha > \alpha_4$  that is excluded because of a singularity being hit in the past. The diamonds denote the maximum likelihood points.

of this analysis. The best fit value in the low region is  $\alpha = 0.9$  and  $\bar{\omega}_m = 0.105$  and in the high region  $\alpha = 2.15$  and  $\bar{\omega}_m = 0.085$ . In order to obtain constraints on the physical matter density  $\omega_m$  and  $\hat{\mu}$  we have to use additional data, which measures the expansion rate today. This can be achieved either with direct measurements of the Hubble rate, like with the Hubble Key Project<sup>21</sup> with  $H_0 = 72 \pm 8$  km/sec/Mpc or with estimates of the age of the Universe via the age of globular clusters<sup>22</sup> with a mean  $t_0 = 13.4$  Gyr with  $t_0$  larger than 11.2 Gyr at the 95% confidence level. In Fig. 2 we show the results for including a prior on  $H_0$ . In this case we obtain  $\omega_m = 0.14 \pm 0.03$  in the low region and  $\omega_m = 0.14 \pm 0.04$  in the high region. An additional constraint we applied is the measurement of the angular diameter distance to the last scattering of cosmic microwave photons. This was given by the WMAP team first year data release to be  $d_A(z = 1100) = 14.0 \pm 0.3$  Gpc. Note that this number has hardly changed with the third year release of the WMAP data<sup>6</sup>. In order to compare the modified gravity models with the constraints from WMAP one can calculate  $d_A(z) = d_L(z)/(1+z)^2$ . However, we want to caution the reader here. In order to use CMB data to constrain modified gravity models one has really to perform a full perturbation analysis for the modified models. Otherwise one can not be sure that the results presented in the dotted lines of Fig. 2 are valid. It might be that the modifications to gravity we propose here are not stable and the whole linear perturbation regime breaks down. This might be a drastic view, but all we want to emphasize is that one

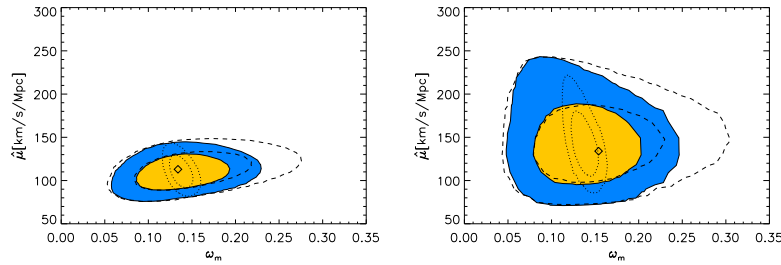


Figure 2: Joint 1- and 2- $\sigma$  contours in  $\omega_m$ - $\hat{H}$ . The large dashed contours are with a prior on  $H_0$  only, filled contours with an additional prior on  $t_0$  and the dotted innermost contours with an additional prior from the angular diameter distance to the CMB last scattering surface as measured by WMAP first year release.

can not be sure before doing such a calculation. Nevertheless the presented constraint can act as a guideline as what is to be expected from CMB constraints if “everything goes well” in a perturbation analysis.

## 5 Conclusion

We have presented an analysis, which shows that an inverse curvature gravity model can explain Supernovae observations of the expansion rate to a satisfactory level. While these models might have many problems regarding their consistent theoretical formulation, one should nevertheless be open minded, that not just an additional component in the Einstein equations, i.e. dark energy, can explain accelerated expansion of the Universe.

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