Understanding High Temperature Superconductivity: Progress and Prospects

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Abstract

From its discovery in 1911 until 1986, it was generally believed that superconductivity could only exist in metals at extremely low temperatures, with a maximum transition temperature for its appearance of some twenty-five degrees above absolute zero. The 1986 discovery of superconductivity at substantially higher temperatures, in materials which were close to being antiferromagnetic, and in which the "action" occurred primarily in planes containing a nearly square array of copper and oxygen atoms, opened a new chapter in physics. Indeed, understanding the appearance of superconductivity at high temperatures (the current maximum transition temperature is 160K) is arguably the major problem in physics today, with over ten thousand researchers working on this topic here and abroad. Following an introduction to the basic concepts of normal metals and conventional, low temperature, superconductivity, I review the experimental results of the past decade, which demonstrate that the high temperature superconductors are strange metals with highly anomalous superconducting properties. I then describe recent theoretical developments which clarify the nature of these strange metals and strongly support the proposal that it is the magnetic interaction between planar quasiparticle excitations which is responsible for their emergent normal state behavior and the appearance of superconductivity at high temperature. The key role played by nonlinear feedback in determining system behavior will be discussed, and the problems which lie ahead will be described.

1 Introduction

In 1911, H. Kamerlingh-Onnes, working in his low temperature laboratory in Leiden, discovered that at a few degrees above absolute zero an electrical current could flow in mercury without any discernable resistance. He named this remarkable new phenomenon, "superconductivity." A theory which explained it was not developed for another forty-six years, when, in 1957, University of Illinois physicists John Bardeen, Leon Cooper, and Robert Schrieffer put forth their microscopic theory, which quickly became known by their initials, as the BCS theory. A third era in superconductivity opened in 1986 when Georg Bednorz and Alex Mueller, working in the IBM Laboratory near Zurich, made another startling discovery, superconductivity at a temperature substantially higher than had hitherto been known, in a class of materials which were completely different from the metals which had previously been found to be superconducting. Their discovery launched a major new field in physics, the study of high temperature superconductivity, or high T_c, as it has become known.

In this lecture, which is intended for the non-specialist, I shall describe how far we have come in understanding high T_c, and discuss the prospects for the development of a microscopic theory. I begin with a review of some basic concepts of the theory of metals, describe some of the steps which led to the BCS theory, and present a BCS primer. I then discuss briefly the developments in our understanding of superconductivity and superfluidity in the universe, developments which were inspired by the BCS theory. These include the discovery of many new classes of superfluid materials, ranging from liquid helium three, which becomes superfluid at a few millidegrees above absolute zero, to neutron matter in the crust of a neutron star, which can become superfluid at temperatures of almost a million degrees. I next discuss the impact of the discovery of high T_c materials, and summarize some key experimental results. I then present a candidate model for high temperature superconductivity, nearly antiferromagnetic Fermi liquid theory, which appears capable of providing a quantitative account of the unusual normal state properties of the highest transition temperature superconductors, the so-called optimally doped materials. I conclude with a tentative explanation for the remarkable normal state properties of the underdoped high temperature superconductors, which represent a fascinating example of a new class of materials, complex adaptive matter, in which intrinsic non-linear feedback, both positive and negative, plays an 1essential role in determining system behavior.

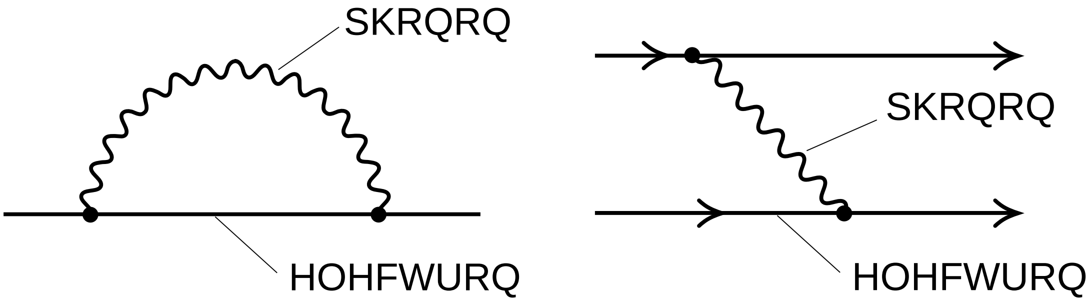
2 Conventional Superconductors: From Discovery to Understanding

In his 1913 Nobel lecture, Kamerlingh-Onnes reported that "mercury at 4.2K has entered a new state, which owing to its particular electrical properties, can be called the state of superconductivity." He noted that the state could be destroyed by applying a sufficiently large magnetic field, while a current induced in a closed loop of superconducting wire persisted for an extraordinarily long time. He demonstrated the latter phenomenon by starting a superconducting current in a coil in his Leiden laboratory, then transporting the coil, plus the "refrigerator" which kept it cold, to Cambridge University for a lecture-demonstration on superconductivity.

It is natural to wonder why superconductivity represented such a difficult problem in physics that forty six years had to pass before it was finally solved. First, for almost twenty years the physics community did not possess the basic building blocks needed to formulate a solution–the quantum theory of normal metals. Second, it was not until 1934 that a key experiment was performed, the demonstration by Meissner that the basic property of a superconductor was its perfect diamagnetism (its ability to shield out an external magnetic field of modest size in a microscopic distance). Third, once the building blocks were in place, it quickly became clear that the characteristic energy associated with the formation of the superconducting state is tiny, roughly a millionth of the normal state characteristic electronic energies. Theorists therefore focussed their attention on developing a phenomenologcal description of superconducting flow. The way was led by Fritz London, who pointed out in 1935 that "superconductivity is a quantum phenomenon on a macroscopic scale, . . . with the lowest energy state separated by a finite interval from the excited states" and that "diamagnetism is the fundamental property."

Let us consider briefly the basic quantum building blocks. First came the recognition that electrons in a metal move in a periodic potential produced by ions which oscillate about their equilibrium positions. The motion of the ions can be described by their quantized collective modes, the phonons. Next, in the course of the development of the quantum theory, came the discovery by Pauli of the exclusion principle which bears his name–that electrons possess a half integral intrinsic spin, and that as a result, no two electrons can possess the same quantum numbers. Particles which possess an intrinsic spin of one-half are known as fermions, in honor of the work of Fermi who, with Dirac, developed the statistical theory of electron behavior at finite temperatures, the Fermi-Dirac statistics. In a momentum space description of a simple metal, the ground state is a sphere in momentum space, whose radius, pf is determined by the electron density. The energy of the outermost electrons, $E_f = p_f^2/2m$ is very large compared to their average thermal energy, kT. As a result, only a fraction of the electrons, kT/E_f, are excited above the ground state. The electrons interact with each other (by Coulomb's law) and with the phonons. Their elementary excitations are quasiparticles, the electrons plus their associated cloud of other electrons and phonons which accompany electrons as they move through the lattice. An elementary argument shows that the lifetime of a quasiparticle excited above the Fermi surface (the surface of the Fermi sphere) is some $(kT)^2/E_f^2$. The problem faced by the theorists was understanding how these interacting electrons could undergo a transition to the superconducting state. What brought it about? What was the appropriate mathematical description?

An essential clue came in 1950, when researchers at the National Bureau of Standards and at Rutgers University discovered that the superconducting transition temperature of lead depended on its isotopic mass, M, being inversely proportional to $M^{1/2}$. Since the lattice vibrational energy displays the same dependence on $M^{1/2}$, their basic quanta, phonons, must somehow play a key role in bringing about superconductivity. In the following year,



(a) (b)

Figure 1: Some consequences of the electron-phonon interaction: (a) a change in the electron self-energy: (b) the phonon-induced electron-electron interaction.

Herbert Frohlich, who was visiting Purdue from his home university of Liverpool, and John Bardeen, who was then at Bell Laboratories, tried and failed to construct a theory based on electron-phonon interaction. What they did can be visualized with the aid of the diagrams introduced by Richard Feynman (the art of the quantum theorist), which are shown in Figure 1(a). There one sees an electron emitting and then absorbing a phonon; its properties are modified by this dynamic coupling to the lattice, and the change in its energy is inversely proportional to $M^{1/2}$. But these quasiparticles do not superconduct.

Frohlich then considered the next possibility, shown in Figure 1(b), where one sees an electron emitting a phonon which is subsequently absorbed by a second electron. This phonon-induced interaction between electrons could be attractive for electrons which are close to the Fermi surface. It is the metallic equivalent of a waterbed; two persons sharing a waterbed tend to be attracted to its center, by the same kind of induced process which attracts the electrons. (One person induces a depression in the bed, a depression into which the second is attracted.) The interaction studied by Frohlich is at first sight quite appealing, being both novel and potentially involving the right dependence on the isotopic mass. There was however a major problem in understanding how it could play a role, since the basic Coulomb interaction between electrons is both repulsive, and very much stronger. As Landau put it, "you can't repeal Coulomb's law." This was the problem which John Bardeen and I attacked, when I was his first postdoctoral researcher at the University of Illinois during the period, 1952-1955. What we found, by extending an approach which David Bohm and I had earlier developed for understanding the consequences of electron-electron interactions in metals, was that "the medium is the message." When we took into account the influence of electronic screening processes on both electron-electron and electron-ion interactions, we found that the presence of a second component, the ions, makes possible a net attractive interaction between a pair of electrons whose energy difference is less than a characteristic phonon energy.

The momentum and frequency dependent effective interaction we found is given in (1)

$$V_{\text{eff}}(\mathbf{q},\omega) = \frac{4\pi e^2}{\mathbf{q}^2 \varepsilon(\mathbf{q},0)} \Big[\frac{\omega^2}{\omega^2 - \Omega_{\mathbf{q}}^2} \Big] \tag{1}$$

where $\varepsilon(\mathbf{q},0)$ is the wavevector dependent static dielectric constant, $\Omega_{\mathbf{q}}$ the phonon energy, $\mathbf{q}$ is the momentum transfer, and ω the difference in electron energies. Its consequences were studied in detail by Leon Cooper, who succeeded me as Bardeen's postdoc in the fall of 1955. He found that because of this net attraction, the normal state Fermi surface could become unstable at low temperatures to the formation of pairs of electrons of opposite spin and momentum. With his work, the solution to superconductivity was near at hand; it came in early 1957, when Bob Schrieffer, who was Bardeen's graduate student at Illinois, realized that a candidate microscopic description of the superconducting state could be developed by applying an approach earlier developed for polarons (by T.D. Lee, Francis Low, and me) to the interacting Cooper pairs. In the ensuing weeks, Bardeen, Cooper, and Schrieffer developed their microscopic theory of superconductivity, the BCS theory, which was so quickly successful at explaining all existing phenomena and predicting new ones, that in June, 1959, at the first major post-BCS conference on superconductivity (held at Cambridge University), David Schoenberg opened the meeting by saying "now, let's see to what extent the experiments fit the theoretical facts."

3 BCS Theory and Its Impact

In BCS theory it is an effective attraction between pairs of electrons of opposite spin and momentum which is responsible for the transition to the superconducting state. Below the superconducting transition temperature, T_c, the pairs form a condensate, a macroscopically occupied single quantum state, which flows without resistance and acts to screen out modest external magnetic fields, thus bringing about the perfect diamagnetism measured in the Meissner effect. At low temperatures, it costs a finite amount of energy, $\Delta \sim 1.75kT_c$, to split up one of the pairs in the condensate; this is the energy gap foreseen by London; its impact on superconducting properties had been worked out phenomenologically by John Bardeen in the years immediately preceeding the development of the microscopic theory. The superconducting state is thus characterized by two distinct components: a superfluid,the condensate, and a normal fluid made up of the single particle excitations which result from the break up of the condensate pairs at finite temperatures. The excited quasiparticles which make up the normal fluid behave display certain coherent effects in response to external fields, coherence phenomena which are a signature of the BCS pairing theory, but otherwise behave normally, in that they collide with one another, with phonons, and with the walls of their container. The characteristic length over which coherent behavior can occur, the coherence length, is of the order of a thousand times the interparticle spacing. To appreciate what is happening, it is instructive to consider the analogy of a dance floor crowded with couples moving to music; in the normal state, the couples collide frequently with each other, while in the superconducting state, those couples which belong to the condensate possess an invisible bond which permits them to glide effortlessly (a la Rogers and Astaire) around the ballroom, even though separated by many intervening couples; it is only the unattached, excited, singles who collide with one another and the walls of the ballroom. The BCS superconducting transition is fundamentally different from what might happen if the pairs had formed well above T_c, and then condensed; in this latter case, the coherence length would be of the order of the interparticle spacing, and the energy gap would not be related to T_c.

BCS theory had a significant impact on many other fields of physics. It predicts that any system of interacting fermions could undergo a superconducting, or in the case of fermions with no charge, a superfluid transition, provided one had a net fermion attractive interaction in some angular momentum channnel. Shortly after the initial publication of the BCS results, Aage Bohr, Ben Mottelson, and I, working together in Copenhagen in the summer of 1957, showed that neutrons or protons in the atomic nucleus would pair as a result of their mutual attraction, and that one could explain in this way many hitherto puzzling nuclear phenomena, while Yoichiro Nambu in Chicago explored the consequences of BCS pairing for the high energy phenomena found in elementary particle physics. The presence of neutron and proton superfluids in the newly discovered pulsars, rotating neuton stars, was invoked in 1969 (by Gordon Baym, Chris Pethick, Mal Ruderman and me) as the explanation for the observed long time decay of the glitches (sudden jumps in the pulsar rotational period) which were discovered in the Vela and Crab pulsars in March and September of 1969. Since 3He atoms are fermions and possess a long range attraction, it was widely expected that liquid 3He would undergo a transition to the superfluid state, and the low temperature physics community searched vigorously for signs of that transition, a search which proved successful for Doug Osheroff, David Lee, and Bob Richardson, of Cornell University, who discovered in 1972 that 3He became a superfluid at a temperature of some three millidegrees above absolute zero.

Needless to say, inspired by the BCS theory, condensed matter experimen-talists sought new classes of superconducting metals, and searched intensively for materials which would become superconducting at substantially higher temperatures than the transition temperatures $\lesssim$ 20K which seemed to characterize normal superconducting metals. Two new classes of superconductors were discovered: the heavy electron materials, $\mathrm{CeCu_2Si_2}$, $\mathrm{UPt_3}$, and $\mathrm{UBe_{13}}$ were found to superconduct at temperatures of about 1K in work carried out by Frank Steglich in Germany, and Zachary Fisk, Jim Smith, and Hans Ott, working at Los Alamos, while Daniel Jerome, in Paris, found superconductivity at temperatures of order 10K in certain nearly two-dimensional organic metals. However, despite the best efforts of Bernd Matthias, who discovered of the order of 100 new superconducting materials, there appeared to be a ceiling to the superconducting transition temperature of approximately 23K, a ceiling which could plausibly be associated with the mechanism responsible for superconductivity, the phonon-induced interaction.

4 The High Temperature Superconductors

A new era in superconductivity opened when, on January 27, 1986, Bednorz and Mueller discovered a sharp drop in the resistance of $La_{2-x}Ba_xCuO_4$ at a temperature of approximately 30K. They sent off a paper reporting their findings to a European journal, the Zeitschrift fur Physik, and continued their study of this novel material in order to be certain that the resistivity change they had observed reflected a transition to the superconducting state. By October they had observed the Meissner effect, and so established that the new material was indeed a superconductor. Word of their results soon spread; a month later, Tanaka and his colleagues in Tokyo confirmed the Bednorz-Mueller results (a confirmation reported in one of Japan's leading newspapers) while their work was further supported by experiments carried out in Beijing by Zhou and his colleagues (whose work was described in the Beijing newspapers that December). In the following month, in a collaborative effort led by Paul Chu of the University of Houston and Mang- Kang Wu of the University of Alabama, a new member of this high temperature superconducting family was discovered, $YBa_2Cu_3O_7$, which possessed a T_c of over 90K. Thus within a year of the original discovery the superconducting transition temperature had increased by a factor of three, and it was clear that a revolution in superconductivity had begun. A celebration of the start of that new era took place at a special evening session of the American Physical Society's 1987 March meeting in New York City, when some 3000 physicists jammed the auditorium in which the session took place, with another 3000 people watching on closed circuit television outside, an event which has become known as the Woodstock of Physics.

Within the next six years a number of additional families of high temperature superconductors were discovered. These included Tl- and Hg- based systems which had maximum T_c's of 120K and 160K respectively. All shared the feature which appeared responsible for the occurrence of high temperature superconductivity, the presence of planes containing Cu and O atoms which are separated by bridging materials which act as charge reservoirs for the planes. During this period, some 10,000 papers a year were being published on high temperature superconductors (a pace which continues to the present time) and it became evident that high temperature superconductivity was regarded by many as the major problem in physics in the last decade of this century. There are at least four reasons for the extraordinary interest in high T_c: its intrinsic scientific interest; its transdisciplinary nature (it reaches across the boundaries which typically divide materials scientists and chemists from experimental and theoretical physicists); the potential applications for materials which superconduct at temperatures greater than the temperature at which nitrogen liquifies (77K), applications which might include filters for cellular phone systems, superconducting transmission lines, MRI machines using high T_c magnets, microwave systems which incorporate the new materials, and hybrid semiconductor/superconductor systems; and finally, the possibility of finding a room temperature superconductor.

Some common characteristics of the high temperature superconductors are that they are ceramic, "flaky" oxides, which are poor metals at room temperature, are difficult materials with which to work. contain few charge carriers compared to normal metals, and display

Table 1: Some ways in which the normal state of high T_c materials is anomalous.

	Conventional	High T_c
Resistivity	$\rho \sim T^2$	$\rho \sim T$
Quasiparticle lifetime, $1/\tau(T,\omega)$	$aT^2 + b\omega^2$	$aT + b\omega$
Spin excitation spectrum	Flat	Peaked at $\mathbf{Q}_i \sim (\pi/a, \pi/a)$
Maximum strength of spin excitations	~ 1 state/eV	$20 - 300$ states/eV
Characteristic spin excitation energy	$\sim E_f$	$\omega_{\rm sf} \sim T \ll E_f$
AF correlations	None	strong, with $\xi_{AF} \geq 2a$
Uniform susceptibility, $\chi_0(T)$	Flat	varies with temperature, possesses a maximum at $T_{cr} > T_c$ for magnetically underdoped systems

highly anisotropic electrical and magnetic properties which are remarkably senstive to oxygen content. While superconducting samples of the 1-2-3 material, $YBa_2Cu_3O_{7-x}$, can be made by a high school student in a microwave oven, single crystals of the high purity required to determine the intrinsic physical properties of these systems are exceedingly difficult to make.

Following a decade of work, there is now an experimental and theoretical consensus that the behavior of the elementary excitations in the Cu-O planes provides the key to understanding the normal state properties of these cuprate superconductors, and that essentially no normal state property (save one) resembles that found in the normal state of a conventional, low T_c, supercon- ductor. As may be seen in Table 1, both the charge response (measured in transport and optical experiments), and the spin response (measured in static susceptibility, nuclear magnetic resonance (NMR) experiments and inelastic neutron scattering (INS) experiments) of the high T_c materials are dramatically different from their low T_c counterparts, as is the single particle spectral density measured in angle-resolved photoemission studies (ARPES).

Moreover, essentially no property of the superconducting state is that of a conventional superconductor, in which BCS pairing takes place in a singlet s-wave state, and the quasiparticle energy gap at low temperatures is finite and isotropic as one moves around the Fermi surface. Despite the fact that something quite new and different is required to understand normal state behavior, there is also a consensus that BCS theory, suitably modified, will provide a satisfactory description of the transition to the superconducting state, and the properties of that state.

There is a near consensus as well on the basic building blocks required to understand the high temperature superconductors. These can be summarized as follows.

- The action occurs primarily in the Cu-O planes, so that it suffices, in first approximation, to focus both experimental and theoretical attention on the behavior of the planar excitations, and to focus as well on the two best-studied systems, the 1-2-3 system ($YBa_2Cu_3O_{7-x}$) and the 2-1-4 system ($La_{2-x}Sr_xCuO_4$).

- At zero doping ($YBa_2Cu_3O_6$; La_2CuO_4) and low temperatures, both systems are antiferromagnetic insulators, with an array of localized Cu^{2+} spins which alternate in sign throughout the lattice.

- One injects holes into the Cu-O planes of the 1-2-3 system by adding oxygen; for the 2-1-4 system this is accomplished by adding strontium. The resulting holes on the planar oxygen sites bond with the nearby Cu^{2+} spins, making it possible for the other Cu^{2+} spins to move, and, in the process, destroying the long range AF correlations found in the insulator.

- If one adds sufficient holes, the system changes its ground state from an insulator to a superconductor.

- In the normal state of the superconducting materials,the itinerant, but nearly localized Cu^{2+} spins form an unconventional Fermi liquid, with the quasiparticle spins displaying strong AF correlations even for systems at doping levels whch exceed that at which T_c is maximum, the so-called overdoped materials.

There is, however, no consensus among theorists as to how to develop a more detailed theoretical description of the cuprates. The approaches which have been tried can be classified as top-down or bottom-up. In a top-down approach, one chooses a model early on (the Hubbard model and the recent SO5 model are typical examples), develops solutions for alternative choices of model parameters, and then sees whether the solutions lead to results consistent with experiment. In a bottom-up approach one begins with the experimental results, and attempts to devise a phenomenological description of a subset of the experimental results. One then explores alternative scenarios which appear consistent with this description, working out the microscopic consequences of each scenario, until one arrives at a scenario and associated microscopic calculations which are consistent with experiment. Then, and only then, does one search for a model Hamiltonian whose solution might provide the ultimate microscopic theory. It was this second approach which John Bardeen followed in his work on conventional superconductors, and guided by his example, it was the approach our research group in Urbana followed for high T_c. We arrived in this fashion at a nearly antiferromagnetic Fermi liquid (NAFL) description of the effective quasiparticle interaction responsible for the strange normal state properties and the superconducting transition at high T_c.

5 Nearly Antiferromagnetic Fermi Liquid Theory

NAFL theory is a bottom-up, experiment-based approach. Since ARPES experiments show that the barely itinerant Cu^{2+} spins possess a well-defined Fermi surface, a Fermi-liquid based approach seems worth pursuing. On the other hand, given the almost bizarre properties found in the normal state, if the planar excitations form a Fermi liquid, that liquid must be really different from the Landau Fermi liquids found in conventional superconductors. One measure of that difference is provided by NMR experiments which show that the optimally-doped 90K superconductor is not far from being antiferromagnetic; it exhibits strong antiferromagnetic correlations between the Cu^{2+} spins (the antiferromagnetic correlation length can exceed two lattice spacings), correlations which become much stronger as one reduces the planar hole concentration or turns to the 2-1-4 system. These two experimental results led us to consider the possibility that the effective planar quasiparticle interactions were such as to drive the system to its nearly antiferromagnetic behavior, and to develop with Andy Millis a phenomenological description of their low frequency magnetic behavior which provided an excellent fit to the NMR experiments. We next made the ansatz that quasiparticle behavior would be determined by a frequency and wavevector dependent effective interaction which was proportional to the low frequency dynamical spin susceptibility, $\chi(\mathbf{q},\omega)$, which provided the fit to NMR experiments,

$$V_{\text{eff}}(\mathbf{q},\omega) = g^2\chi(\mathbf{q},\omega) = g^2\frac{\chi_Q}{1+(\mathbf{Q}-\mathbf{q})^2\xi^2 - i\omega/\omega_{\text{sf}}} \tag{2}$$

Here χ_Q is the static susceptibility at the commensurate wavevector, $\mathbf{Q} = (\pi,\pi)$ which characterizes antiferromagnetic behavior in the insulator, ξ is the antiferromagnetic correlation length, and ω_{sf} is the frequency of the spin fluctuation relaxational mode. Because the systems of interest display near antiferromagnetic behavior, one finds quite generally that

$$\chi_Q \gg \chi_0 \tag{3a}$$

$$\xi \gg a/\pi \tag{3b}$$

$$\omega_{\text{sf}} \ll E_f \tag{3c}$$

where the quantities on the right hand side of (3a) to (3c) represent the corresponding "normal" Fermi liquid values for these parameters.

Our proposed magnetic quasiparticle interaction is thus highly peaked in momentum space, with the interaction being very strong for quasiparticles located a distance, $\mathbf{Q}$, away from one another on the Fermi surface, and comparatively weak for the quasiparticles located more than an inverse correlation length away from those quasiparticles which feel the maximum effective interaction. Such a highly anisotropic quasiparticle interaction is very different from the comparatively featureless quasiparticle interaction encountered in normal superconducting metals; the key question was whether it could give rise to the anomalous normal state behavior and high T_c found in the superconducting cuprates.

To answer this, our research group in Urbana (which included the highly talented graduate students, Philippe Monthoux, Dean Thelen, and Victor Barzykin, and postdocs, Hartmut Monien, Alexander Balatsky, Joerg Schmalian, Alexander Sokol, and Branko Stojkovic) has carried out microscopic calculations of a number of normal state properties as well as of the superconducting transition temperature. For a given system, our calculations involved only one free parameter, the coupling constant g, since both the starting quasiparticle spectra and the frequency and momentum dependence of the candidate effective interaction were taken from fits to experiment. The results of these calculations, carried out over a seven year period (1990-1997) may be summarized as follows:

- In both weak and strong coupling calculations, the calculated resistivity at high temperatures was linear in T, as is seen experimentally.
- For a coupling constant, g, which yielded quantitative agreement with experiment for the optimally doped system, $\mathrm{YBa_2Cu_3O_7}$, we found, in a strong coupling (Eliashberg) calculation, that the transition to superconductivity occurred at 90K, as is seen experimentally.
- The pairing state which characterizes superconductivity was, however, quite different from the singlet "s" pairing state of the conventional superconductors. It is called $d_{x^2-y^2}$ which means that the energy gap takes the form

  $$\Delta(k,T) = \Delta(T)\left[cos(k_x a) - cos(k_y a)\right] .$$

  It thus varies as one goes around the Fermi surface, and vanishes whenever $k_x^2 = k_y^2$.
- The high degree of quasiparticle anisotropy arising from the strong peaks in momentum space in the quasiparticle interaction provided a natural explanation for the measured variation with temperature and doping of the transverse conductivity (the Hall effect) and the optical properties.

When Philippe Monthoux and I found that we could explain two distinct properties of the optimally doped 1-2-3 system with a single free parameter, we concluded that we had a "proof of concept" for the NAFL approach, and more generally, that our work strongly supported the magnetic, or spin fluctuation origin of high temperature superconductivity. However at the time of our work in 1991, most members of the high T_c community believed that the pairing state was not the one our theory required, but rather was the "s" state found in conventional superconductors, since of the many different experiments which had been used to probe the nature of the pairing, only one NMR experiment appeared consistent with our pairing assignment. Since we believed that our theory incorporated so much of the right physics, and possessed predictive power, we therefore challenged the experimental community to prove us right or wrong; we promised to withdraw our theory should subsequent experiments show anything other than the pairing state our theory unambigiously predicted. Fortunately (for us, anyway) our prediction turned out to be right.

By March, 1993, some five experiments (3 distinct NMR results, one penetration depth, one tunneling experiment) supported our pairing state, while by March, 1995 there were some 43 experiments which strongly, or uniquely, supported $d_{x^2-y^2}$ pairing. There were perhaps three reasons for this remarkable change in the experimental conclusions. The first was the use of much purer samples, coupled with the realization that for a d-wave superconductor, impurities and imperfections, when present, could give rise to a false "s-wave-like" signature. The second was the SQUID experiment of Dale van Harlingen and his student David Wollman, who used samples prepared by their Urbana colleague, Don Ginsberg, to show that one could, in suitable geometry, arrive at an unambigious determination of the order parameter,as had been suggested by Tony Leggett, Maurice Rice, and Manfred Sigrist. The third reason was that a number of experimenters were stimulated by our proposal that the mechanism for high T_c could be established through a measurement of the symmetry of the energy gap, or order parameter. What they, and we, had failed to anticipate was that as soon as the symmetry of the order parameter had been established, proponents of alternative mechanisms would find a way (albeit occasionally tortuous) to obtain that symmetry using their mechanisms of choice.

There is a simple physical reason why the NAFL model always yields $d_{x^2-y^2}$ pairing. When one examines the character of a magnetic interaction which mirrors the peaks in the spin fluctuation spectrum required to explain the NMR experiments, one sees that in configuration space, the effective interaction between the almost localized quasiparticles will be strongly repulsive for particles which attempt to occupy the same position, attractive for the four nearest neighbor quasiparticles, then repulsive for next-nearest neighbors,etc., in such a way that it is always repulsive along the diagonals of the "lattice." For the $d_{x^2-y^2}$ state, the nodes of the energy gap are located along the diagonals, passing of course through the origin, so that the effective repulsion present in the NAFL model does not interfere with the pairing brought about by the nearest neighbor attraction. This simple physical model also explains a second result which Monthoux and I had found, that when the AF correlation length was less than the interparticle spacing, T_c plummeted; this comes about because for such short correlation lengths the quasiparticles do not sample the nearest neighbor attraction found systems with stronger AF correlations; since there is no longer a natural source of attraction, T_c must be markedly smaller.

There are two normal state signatures of NAFL behavior:

- Hot and cold quasiparticles
- The appearance of a hot quasiparticle pseudogap in the magnetically underdoped systems.

As shown in Figure 2, a quasiparticle interaction which mirrors the peak structure found in NMR and INS experiments produces a "two-class society"; hot quasiparticles (the "elite") interact very strongly because they feel the peaks in the effective magnetic interaction; cold quasiparticles (the "underclass")) feel only the valleys, a normal Fermi liquid kind of interaction. The resulting anisotropy in quasiparticle behavior (for example the lifetime of a cold quasiparticle is very much longer than that of a hot quasiparticle) as one moves around the Fermi surface explains the measured anomalous transport and optical behavior of the optimally doped or overdoped systems. As may be seen in Figure 3, a direct analysis of experiments on the longitudinal and transverse conductivities of single crystals yields distinct hot and cold quasiparticle lifetimes which agree well with the NAFL calculations.

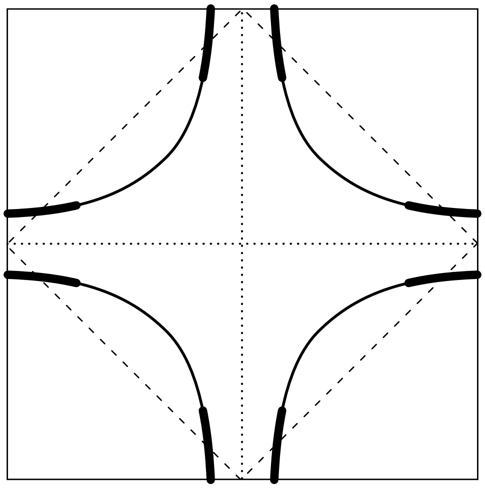


Figure 2: Hot (thick lines) and cold (thin lines) quasiparticles on the Fermi surface. Hot quasiparticles on the Fermi surface, which lie a distance apart in momentum space of $(\mathbf{Q} \pm 1)/\xi$, feel the maximum consequences of the NAFL interaction, (1); the remaining (cold) quasiparticles are much more weakly coupled.

For the magnetically underdoped systems, the exceptionally strong interaction between the hot quasiparticles leads to a change in their character; below a characteristic temperature, which corresponds roughly to that temperature at which the correlation length is equal to twice the lattice spacing,and the uniform spin susceptibility takes on its maximum value, there is a transfer of the quasiparticle spectral weight from low to high frequencies, as though a gap had opened up in the hot quasiparticle spectrum. This pseudogap behavior, which has been calculated very recently by Jorge Schmalian, is seen in ARPES, specific heat, Raman scattering, and uniform susceptibility experiments, as well as in the NMR measurements of the spin-lattice relaxation rate and the spin-echo decay rate.

Now that the symmetry of the pairing state has been established, what are the "frontier" problems in high T_c? One is understanding the doping and temperature dependence of the quasiparticle pseudogap behavior found in the magnetically underdoped cuprates, which leads to three distinct phases in the normal state, that is, at temperatures above T_c. Candidate phase diagrams for two high T_c systems are shown in Figure 4. For both, above T_{cr}, which marks the maximum in the uniform susceptibility, one gets mean field behavior

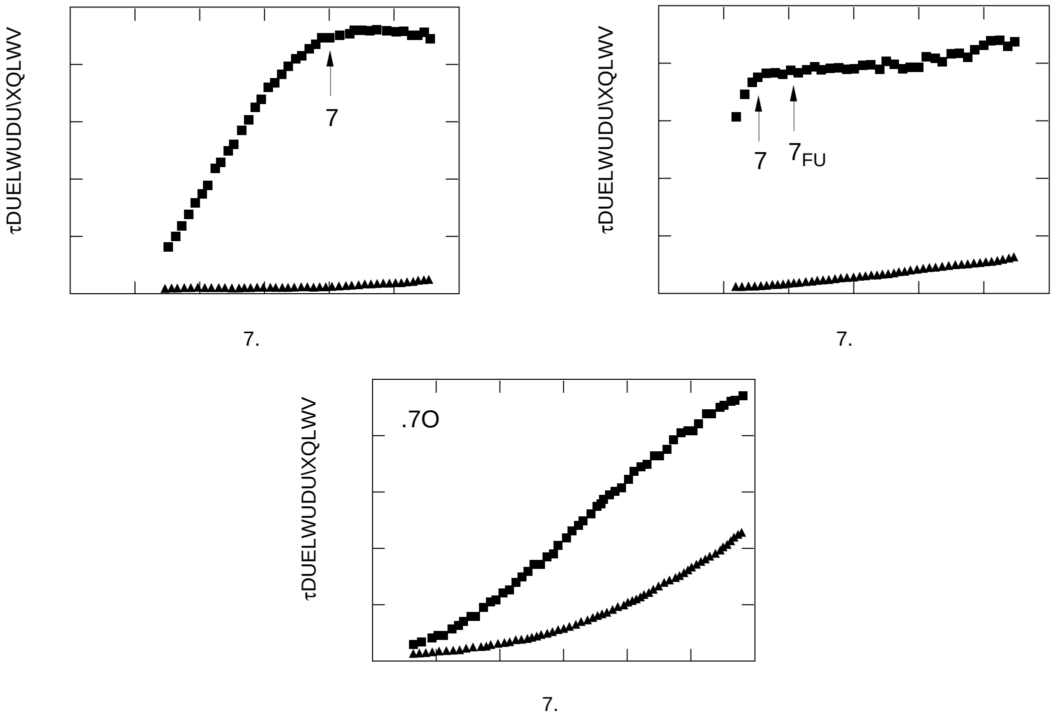


Figure 3: Hot and cold quasiparticle lifetimes, deduced from measurements of the resistivity and Hall effect, for three cuprate superconductors (after Stojkovic and Pines); the squares denote the hot quasiparticles. The crossover at T_* found in the magnetic behavior of the underdoped systems is clearly visible in the 1-2-3 $O_{6.68}$ and 1-2-3 O_7 results.

for the spin fluctuation spectrum, the quasiparticle spectrum is sharply peaked at the Fermi surface, and strong coupling (Eliashberg) calculations provide a quantitative understanding of system behavior. At T_{cr} one gets a crossover in system behavior, to a regime (the weak pseudogap or pseudoscaling regime) in which the peak in the hot quasiparticle spectrum becomes broad and is shifted to much higher energies, while the relationship between the characteristic spin fluctua-tion energy and the correlation length changes character. A second crossover occurs at a still lower temperature, T_*; below this temperature one is in the strong pseudogap regime. The hot quasiparticles develop the leading edge gap measured in ARPES and Raman scattering experiments, while the AF correlation length becomes frozen, and the spin fluctuation energy, ω_{sf}, takes on a very different temperature dependence.

The presence of these two crossover temperatures is perhaps not surprising in a system in which the quasiparticle interactions are of electronic origin, so that the system displays intrinsic non-linear behavior. The planar quasiparticles are both responsible for their mutual interaction and change their behavior in response to that interaction, as illustrated

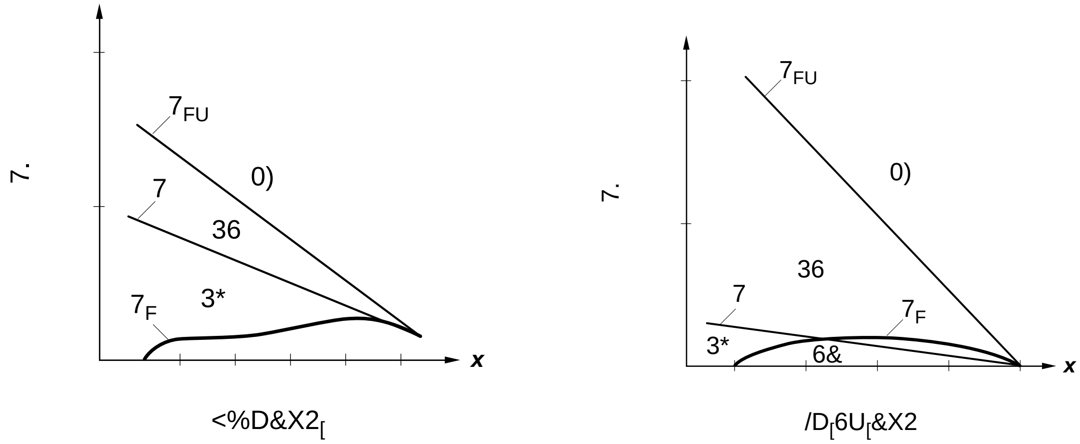


Figure 4: Candidate phase diagrams for two families of cuprate superconductors. In both the 1-2-3 and 2-1-4 systems one finds in NMR experiments the crossovers at T_{cr} and T_* from mean-field (MF) to pseudoscaling (PS) to pseudogap (PG) behavior discussed in the text, before the transition at T_c to the superconducting (SC) state. Note the similarities in the doping dependence of T_{cr}, T_*, and T_c.

schematically below

$$\begin{array}{c}\text{Interaction} \longrightarrow \text{quasiparticle behavior}\\ \text{Quasiparticle behavior} \longrightarrow \text{interaction}\end{array}$$

The resulting feedback loop can be either negative (tending to maintain the status quo) or positive (tending to bring about dramatic changes); negative feedback explains the system behavior above T_{cr}; positive feedback is responsible for the crossovers at T_{cr} and T_*.

From this perspective, superconducting cuprates are an intensively studied example of systems in which intrinsic non-linear behavior brings about dramatic changes in system dynamics in response to small changes in doping levels, temperature, applied external fields, etc. Such systems have been receiving increased attention in the condensed matter and materials science communities and are perhaps best described by the phrase, complex adaptive matter. Other examples of complex adaptive matter are spin glasses, heavy electron systems, materials which display colossal magnetoresistance, and the protein matter of interest to the biological physics community.

To sum up, I have presented in this lecture answers to some of the key questions about high T_c. The physical origin of the anomalous normal state behavior is the highly anisotropic effective magnetic interaction between the almost localized planar quasiparticles, which are hybrids of holes and localized Cu^{2+} spins. The normal state is best described as a nearly antiferromagnetic Fermi liquid. The mechanism for high T_c is spin-fluctuation exchange, an electronic mechanism, which produces a quasiparticle interaction which mirrors the dynamic spin susceptibility measured in NMR experiments. The superconducting order parameter and pairing state is the $d_{x^2-y^2}$ state.

We are, however, far from possessing a complete understanding of these fascinating materials. For example we do not have as yet microscopic calculations of the strong pseudogap behavior found below T_*, or of the doping dependence of either T_* or the transition temperature, T_c. Let me close then with a suggested high T_c program for the millenium.

- Microscopic calculations of the planar quasiparticle effective interaction and spectral density for both overdoped and underdoped systems, coupled with
- Benchmark transport, ARPES, magnetic resonance, and neutron scattering experiments on the same representative members of overdoped and underdoped systems.
- Combining theory and experiment to understand in detail the transition from AF insulator to superconductor to normal metal as one varies the planar hole density.
- Determining the maximum T_c achievable with the spin-fluctuation-exchange mechanism.
- Examining other electronic mechanisms in the hope that one might yield a still higher T_c.
- Is room temperature superconductivity possible?

6 Acknowledgements

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